

## Quasi-Steady Droplet Combustion with Fuel Leakage

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### **Abstract.**

*This work addresses the soot formation problem in diffusion flames using the flame fuel leakage model developed for the droplet combustion. From the energetic point of view, soot and pollutant gases carry chemical energy not converted to heat in the flame to the ambient; the same occurs with flame fuel leakage. The parallel between soot and fuel leakage opens opportunities of using results from flame fuel leakage model to understand the effect of soot formation on the diffusion flame properties. To gain insight and to reduce the complexity of the problem, the model is based on some simplifying assumptions. One of them is reaction to be performed through a one-step mechanism with large activation energy, under which asymptotic expansion is a valid strategy to analyse the problem. The results show that the fuel leakage model can be used to describe the soot formation, but some droplet combustion properties are not well represented by the model.*

**Keywords:** droplet combustion, diffusion flame, large activation energy asymptotics

### **1. Introduction**

Liquid fuels need to be atomized in very small droplets for being efficiently burned. The burning condition imposes some important simplification on the description of isolated droplet combustion. The buoyancy force and the relative velocity between the droplet and the gas phase are negligible small, because that force is proportional to the droplet volume and that velocity is inversely to the droplet mass. Thereby, the processes in the gas phase around the droplet presents spherical symmetry. Moreover, due to the large thermal inertia of the liquid phase in normal pressures, those processes are quasi-steady compared to the liquid phase processes, e.g. the heating and vaporisation of the droplet (1; 2). Although, the fluid dynamical aspect of the droplet combustion is relatively simple (1; 2), the chemical aspect and its influence on the flow field are complex. This work addresses, particularly, the description of the flame history for very small stoichiometric ratio of fuel and air to apply the results in benefit of understanding soot formation effect on diffusion flames.

It is well known that fuels with very small stoichiometric fuel-air ratios do not burn completely (3; 4; 5; 6; 7; 8; 9; 10). The time that those fuels spend inside the flame is not enough to complete the oxidation reaction. Then, soot and pollutant gases are formed. From the energetic point of view, soot and pollutant gases are species that flow by the flame and contain chemical energy not converted to heat in the flame.

The classical model for the quasi-steady droplet combustion predicts that the flame position takes place very far from the droplet surface for very small stoichiometric ratio of fuel and air (11; 12). However, the experimental observation does not confirm the theoretical prediction (10). A possible reason for that is the soot formation. The soot can be interpreted as fuel unburned leaking by the flame to the atmosphere. Under this condition it is expected the flame to take place at a different position, closer to the droplet surface.

A model for soot and soot-shell formation was presented (13). The thermophoretic force on the soot particles and the thermal radiative heat loss are included. The results for flame standoff distance and the soot-shell standoff distance agreed very well with the experiments. This is an indication that the soot chemistry is a responsible for discrepancy between the classical results and the experimental results.

To gain insight of the effect of the incompleteness of reaction on the droplet combustion, this work will analyse and trace a parallel between an idealised situation of fuel leakage by the flame and the soot and pollutant gases emissions. At this first approach, the reaction is supposed to perform by a one-step mechanism. Also, it is admitted that the activation energy of that reaction is much larger than the gas phase thermal energy. Under these conditions, the transport phenomena and the fuel reaction can be described by the large activation energy asymptotics.

### **2. Mathematical Formulation**

The formulation of quasi-steady droplet combustion and the solution are presented elsewhere (14). Thus, because of the problem to be well known, only the parts of the formulation essential for the understanding will be explicitly presented.

Considering the ambient conditions to be characterised by the temperature  $T_\infty$ , density  $\rho_\infty$ , oxygen mass fraction  $Y_{O_\infty}$ . Without losing any important feature of the problem, the constant pressure specific heat is supposed constant, but the transport coefficients (thermal conductivity and diffusion coefficient, respectively) are dependent only on temperature according to  $K/K_\infty = D_i/D_{i\infty} = (T/T_\infty)^n = \theta^n$ . The nondimensional quasi-steady conservation equations, describing

the gas phase around the droplet with radius  $a$  at time  $t$  ( $a_0$  at the time  $t = 0$ ), are expressed by

$$x^2 \varrho v = \lambda(\tau) \quad (1)$$

$$\frac{\lambda}{x^2} \frac{\partial y_i}{\partial x} - \frac{1}{x^2} \left( \frac{x^2 \theta^n}{Le_i} \frac{\partial y_i}{\partial x} \right) = -s_i \omega, \quad i=O,F \quad (2)$$

$$\frac{\lambda}{x^2} \frac{\partial \theta}{\partial x} - \frac{1}{x^2} \left( x^2 \theta^n \frac{\partial \theta}{\partial x} \right) = q\omega, \quad (3)$$

The definition of the nondimensional independent variables are as following: the time  $\tau = t/t_c$ , where  $t_c$  is the vapourisation time  $t_c = \varepsilon(a_0^2/\alpha_\infty)$  and the radial coordinate  $x = r/a_0$ ;  $\varepsilon = \rho_\infty/\rho_l$  is the ratio of the gas density to the liquid density and  $\alpha_\infty = K_\infty/(\rho_\infty c_p)$  is the thermal diffusivity. The definition of the nondimensional dependent variables (temperature, density, oxygen mass fraction, fuel mass fraction and velocity, respectively) are as following:  $\theta = T/T_\infty$ ,  $\varrho = \rho/\rho_\infty$ ,  $y_O = Y_O/Y_{O\infty}$ ,  $y_F = Y_F$  and  $v = Va_0/\alpha_\infty$ . The parameters that appearing in Eqs. (2) and (3) are defined as: Lewis number  $Le_i = \alpha_\infty/D_{i\infty}$ ,  $s = s_O = \nu/Y_{O\infty}$ ,  $s_F = 1$  and  $q = Q/(c_p T_\infty)$ . Since the kinetic mechanism is supposed to be one-step and the reaction rate follows Arrhenius type, thus

$$\omega = \frac{Ba_0^2 \rho_\infty^{n_1+n_2-1}}{\alpha_\infty W_O^{n_1} W_F^{n_2}} y_O^{n_1} y_F^{n_2} \exp(-\theta_a/\theta) \quad (4)$$

where  $W_i$  is the molecular weight of species  $i$  and the nondimensional activation energy is defined by  $\theta_a = E/RT_\infty$ .

Equations (2) and (3) are integrated from the droplet surface to the ambient atmosphere, the flame is at a position between these two boundaries. To proceed the integration, the boundaries conditions must be specified at  $x = a = a/a_0$ ,

$$-\frac{x^2 \theta^n}{Le_F} \frac{\partial y_F}{\partial x} = \lambda(1 - y_{Fs}), \quad x^2 \theta^n \frac{\partial \theta}{\partial x} = \lambda l + q^- = \lambda l', \quad \theta = \theta_s, \quad y_F = y_{Fs} = \exp[\gamma(1 - \theta_B/\theta_s)] \quad (5)$$

and for  $x \rightarrow \infty$ ,

$$\theta - 1 = y_O - 1 = y_F = 0 \quad (6)$$

The subscript  $s$  represents the droplet surface condition. The nondimensional latent heat  $l$  is expressed by  $L/(c_p T_\infty)$ ,  $q^-$  is the heat to inside the droplet and  $l' = l + q^-/\lambda$ .

In this work, it is admitted the temperature profile is uniform inside the droplet, but it varies with time,  $\theta = \theta_s(\tau)$ . Thereby, the mass conservation equation for the liquid phase leads to

$$\frac{da^2}{d\tau} = -2 \frac{\lambda}{a} \quad (7)$$

The definition of the nondimensional vaporisation rate is  $\lambda = \dot{m}/(4\pi a_0 K_\infty/c_p)$  and the nondimensional droplet radius,  $a = a/a_0$ . The ratio  $\lambda/a$ , known as vaporisation constant and defined as  $\beta$ , depends on the heat flux to the droplet imposed by the flame.

The closure for the system of equations is provided by the dimensionless equation of state of the gas,  $\varrho \theta = 1$ .

The system of Eqs. (1) to (3) is characterised by the boundary conditions Eqs. (5) and (6).

According to the type of the problem, at the flame  $x = x_p$ , the properties are

$$\theta - \theta_p = y_F - y_{Fp} = y_O = 0 \quad (8)$$

In this analysis, the fuel leakage  $y_{Fp}$  through the flame without reacting is supposed to be of the order of  $Le_F/Le_{Os} = 1/S \ll 1$ . To distinguish the results with fuel leakage from those without leakage, the subscript  $p$  will be used to identify the leakage and  $f$  to identify no leakage.

## 2.1 Flame Outer Zone Description

In first approximation, the flow generated by the phase change and thermal expansion is described by Eqs. (1-8) imposing the hypothesis of infinite thin flame with oxygen concentration going to zero at the flame and the fuel concentration going to a fix value. Under the fuel leakage condition, the flow field analysis does not provide a closed solution. The flame properties (position and temperature of the flame) and the droplet properties (droplet temperature and vaporisation rate) are determined as function of the fuel leakage quantity.

Even with fuel leakage, the Shvab-Zeldovich formulation is the appropriated strategy to solve this problem, by using the fact that there is no oxygen leakage by the flame,  $y_O = 0$  in the part of the domain  $a < x < x_p$ . For these conditions, the Shvab-Zeldovich formulation with the excess enthalpy  $H = (S + 1)L_F \theta/q + y_F + y_O = (S + 1)L_F \theta/q + y_F$  and

the mixture fraction  $Z = Sy_F - y_O + 1 = Sy_F + 1$  can be used to determined the temperature and fuel profiles between the droplet surface and the flame,  $a < x < x_p$ . From previous analysis (14),

$$\lambda(H + H_a) - x^2\theta^n \frac{\partial H}{\partial x} + \frac{(L_F - 1)}{S}\lambda(Z_a - Z) = 0 \quad (9)$$

$$-L_F\lambda(Z_a - Z) - x^2\theta^n \frac{\partial Z}{\partial x} = 0 \quad (10)$$

where  $H_a = [(S + 1)L_F/q](l' - \theta_s) - 1$ ,  $Z_a = S + 1$  and  $S = Le_{O2}/Le_F$  is the ratio of the oxygen flux to the fuel flux at the flame.

The equations for  $H$  and  $Z$  satisfy the boundary conditions at the droplet surface  $x = a$ , which are determined from Eq. (5),

$$H(a) = (S + 1)L_F\theta_s/q + y_{Fs}, \quad Z(a) = Sy_{Fs} + 1, \quad (11)$$

$$x^2\theta^n \left. \frac{\partial H}{\partial x} \right|_{x=a} = \frac{(S + 1)L_F}{q}\lambda l' - \lambda Le_F(1 - y_{Fs}), \quad x^2\theta^n \left. \frac{\partial Z}{\partial x} \right|_{x=a} = -\lambda Le_F S(1 - y_{Fs}) \quad (12)$$

at the flame, the boundary conditions are given from Eq. (6),

$$H(x_p) = (S + 1)L_F\theta_p/q + y_{Fp}, \quad Z(x_p) = Sy_{Fp} + 1, \quad (13)$$

and from the ambient atmosphere  $x \rightarrow \infty$

$$H(\infty) = (S + 1)L_F/q + 1, \quad Z(\infty) = 0, \quad (14)$$

Solving Eqs. (10) and (11) and writing the solution in terms of temperature and fuel mass fraction, it is found (14)

$$\theta = \theta_s - l' + l' \left( \frac{1 - y_F}{1 - y_{Fs}} \right)^{1/L_F} \quad (15)$$

The mixture fraction  $Z$  and the excess enthalpy  $H$  in the part of the domain  $x > x_p$  do not determine the temperature and fuel and oxygen profiles. The reason for that is the presence of the fuel in  $x > x_p$ , making the system of equation undetermined. Of coarse, either one of the unknown must be found from its conservation equation or two unknowns must be written as a function of the third unknown. Despite the indetermination of the problem using  $H$  and  $Z$ , equations for  $H$  and  $Z$  can be integrated once,

$$\lambda(H - H(\infty) + q_H) - x^2\theta^n \frac{\partial H}{\partial x} + (L_O - 1)\lambda(y_O - 1) + (L_F - 1)\lambda y_F = 0 \quad (16)$$

$$\lambda(Z + q_Z) - x^2\theta^n \frac{\partial Z}{\partial x} - (L_O - 1)\lambda(y_O - 1) + S(L_F - 1)\lambda y_F = 0 \quad (17)$$

where  $q_H = x^2\theta^n (\partial H/\partial x)|_{x \rightarrow \infty}$  and  $q_Z = x^2\theta^n (\partial Z/\partial x)|_{x \rightarrow \infty}$  which are determined by the continuity of the functions  $H$  and  $Z$  and their first derivative at the flame  $x = x_p$ ,  $q_H = [(S + 1)L_F/q](1 + l' - \theta_s) + L_O - L_F + (L_F - 1)y_{Fp}$  and  $q_Z = -L_O(s + 1)$ , respectively.

As considered initially,  $1/S \ll 1$ , a solution in this part of the domain is found expanding the those properties in series where the small parameter is  $S^{-1}$ . Using the  $\theta = \theta_0 + S^{-1}\theta_1 + o(S^{-1})$ ,  $y_O = y_{O0} + S^{-1}\psi_O + o(S^{-1})$ ,  $y_F = S^{-1}\psi_F + o(S^{-1})$ , the equations for  $H$  and  $Z$  in the order unity are expressed as

$$\lambda(H_0 - H(\infty) + q_{H0}) - x^2\theta^n \frac{\partial H_0}{\partial x} + (L_O - 1)\lambda(y_{O0} - 1) = 0 \quad (18)$$

$$\lambda(Z_0 + q_Z) - x^2\theta^n \frac{\partial Z_0}{\partial x} - (L_O - 1)\lambda(y_{O0} - 1) + (L_F - 1)\lambda\psi_F = 0 \quad (19)$$

where  $H_0 = (S + 1)L_F\theta_0/q + y_{O0}$ ,  $Z_0 = \psi_F - y_{O0} + 1$  and  $q_{H0} = [(S + 1)L_F/q](1 + l'_0 - \theta_{s0}) + L_O - L_F$

It is worth to note that,  $H_0$  as  $Z_0$  are combination of two variables, and the oxygen mass fraction  $y_{O0}$  is present in both functions. Then, writing both equations in terms of  $y_{O0}$  and integrating, the solutions are (14)

$$\theta_0 = 1 + (\theta_{s0} + q - l'_0 - 1) \left[ 1 - \left( \frac{s + y_{O0}}{s + 1} \right)^{1/L_O} \right] \quad (20)$$

The flame temperature for without fuel leakage through the flame is determined imposing the condition  $y_{O_0} = 0$  and  $\theta_0 = \theta_f$ , thus

$$\theta_f = 1 + (\theta_{s0} + q - l'_0 - 1) \left[ 1 - \left( \frac{s}{s+1} \right)^{1/L_O} \right] \quad (21)$$

Considering the droplet temperature equal to the boiling temperature  $\theta_{s0} = \theta_B$ , then  $l'$  is equal to the latent heat  $l$ ,  $l' = l$ . From Eqs. (15) and (21), the flame temperature with fuel leakage is

$$\theta_p = \theta_f - \frac{1}{L_F} \frac{l}{(1 - y_{F_s})^{1/L_F}} \frac{\psi_{F_p}}{S} \quad (22)$$

and the fuel mass fraction at the droplet surface in the gas phase side is specified by

$$\frac{l}{(1 - y_{F_s})^{1/L_F}} = q + (1 + l - \theta_B - q) \left( \frac{s}{s+1} \right)^{1/L_O} \quad (23)$$

Integrating the energy conservation equation, Eq. (3), the vaporisation constant  $\beta = \lambda/a$  and the flame standoff distance  $x_p/a$  are determined as following

$$\beta = \int_{\theta_B}^{\theta_p} \frac{x^n}{x + l - \theta_s} dx + \int_{\theta_p}^1 \frac{x^n}{x + l - \theta_s - q} dx \quad (24)$$

$$\frac{x_p}{a} = \beta / \left( \int_{\theta_p}^1 \frac{x^n}{x + l - \theta_s - q} dx \right) \quad (25)$$

These properties can be obtained considering the approximation  $\beta = \beta_0 + S^{-1}\beta_1 + o(S^{-1})$  and  $x_p/a = (x_p/a)_0 + S^{-1}(x_p/a)_1 + o(S^{-1})$ , where

$$\beta_0 = \int_{\theta_B}^{\theta_f} \frac{x^n}{x + l - \theta_s} dx + \int_{\theta_f}^1 \frac{x^n}{x + l - \theta_s - q} dx \quad (26)$$

$$\beta_1 = - \frac{l\psi_{F_p}}{L_F(1 - y_{F_s})^{1/L_F}} \left( \frac{\theta_f^n}{\theta_f + l - \theta_s} + \frac{\theta_f^n}{\theta_f + l - \theta_s - q} \right) \quad (27)$$

$$\left( \frac{x_p}{a} \right)_0 = \frac{x_f}{a} = \beta_0 / \left( \int_{\theta_f}^1 \frac{x^n}{x + l - \theta_s + q} dx \right) \quad (28)$$

$$\left( \frac{x_p}{a} \right)_1 = \beta_1 \left( \int_{\theta_f}^1 \frac{x^n}{x + l - \theta_s + q} dx \right)^{-1} - \frac{l\psi_{F_p}}{L_F(1 - y_{F_s})^{1/L_F}} \frac{\beta_0 \theta_f^n}{\theta_f + l - \theta_s - q} \left( \int_{\theta_f}^1 \frac{x^n}{x + l - \theta_s - q} dx \right)^{-2} \quad (29)$$

As can be seen, the droplet combustion properties are determined as a function of the fuel leakage  $\psi_{F_p}$ . According to previous analysis (15), the value of  $\psi_{F_p}$  is a function of the Damköhler number of the reaction. The relation between  $\psi_{F_p}$  the Damköhler number is found when the flame structure is described. The next order problem, in terms of the reciprocal of the activation energy, will treat the issues concerning the flame inner zone. From the flame outer zone, the heat fluxes and the fuel and oxygen fluxes are passed to the inner zone as boundary conditions. These properties are determined by Eqs. (2), (9), (10), (16), (17),

$$d_F \equiv \frac{\partial y_F}{\partial x} \Big|_{x=x_p^-} = \frac{L_F \lambda}{x_p^2 \theta_p^n} (-1 + S^{-1} \psi_{F_p}), \quad d_O \equiv \frac{\partial y_O}{\partial x} \Big|_{x=x_p^+} = \frac{L_F S \lambda}{x_p^2 \theta_p^n} (1 - S^{-1} \psi_{F_p}), \quad (30)$$

$$d_\theta^- \equiv \frac{\partial \theta}{\partial x} \Big|_{x=x_p^-} = \frac{\lambda}{x_p^2 \theta_p^n} (\theta_p + l - \theta_B), \quad d_\theta^+ \equiv \frac{\partial \theta}{\partial x} \Big|_{x=x_p^+} = \frac{\lambda}{x_p^2 \theta_p^n} (\theta_p + l - \theta_B - q). \quad (31)$$

Also, the flame temperature and flame position, given by Eqs.(22) and (25), are necessary to describe the flame structure.

## 2.2 Flame Inner Zone Description

The description of the flame structure is conveniently expressed by the following variables

$$\begin{aligned}\theta &= \theta_p - \varepsilon(\Theta + \gamma\xi + p) + o(\varepsilon) \\ y_O &= \varepsilon d_O \Phi_O / m + o(\varepsilon) \\ y_F &= y_{Fp} + \varepsilon d_F \Phi_F / m + o(\varepsilon) \\ x &= x_p + \varepsilon(\xi + p/\gamma)/m + o(\varepsilon)\end{aligned}\quad (32)$$

where the small parameter  $\varepsilon$  is defined as  $\varepsilon = \theta_p^2/\theta_a$ .

Taking Eq.(32) into Eqs. (2) and (3), the system of equations that describes the flame, is

$$\frac{d^2\Theta}{d\xi^2} = \frac{d^2\Phi_O}{d\xi^2} = \frac{Da}{\Gamma(1+n_1)} \Phi_O^{n_1} \exp[-(\Theta + \gamma\xi + p)] \quad (33)$$

where the definition of the Damköhler number  $Da$  is

$$\begin{aligned}Da &= \Delta \left(\frac{x_p/a}{x_f/a}\right)^4 \left(\frac{a}{\beta}\right)^2 \left(\frac{\theta_p}{\theta_f}\right)^{2+n+n_1-n_2} \frac{y_{Fp}^{n_2}}{(1-y_{Fp})^2} \exp\left(-\frac{\theta_a}{\theta_p} + \frac{\theta_a}{\theta_f}\right) \\ \Delta &= \Gamma(1+n_1) \frac{Ba_0^2}{\alpha_\infty \theta_a^{1+n_1}} \frac{\rho_\infty^{n_1+n_2-1}}{W_O^{n_1} W_F^{n_2}} \frac{(L_F S)^{n_1}}{q^{1+n_1}} \left(\frac{x_f}{a}\right)^4 \theta_f^{2+n+n_1-n_2} \exp\left(\frac{-\theta_a}{\theta_f}\right)\end{aligned}$$

where the conservation of species and energy around the flame,  $(d_F^+ - d_F^-)/L_F = d_O/(L_F S) = (d_\theta^- - d_\theta^+)/q = m/q$ , and Eqs. (22) and (32) were used.

The expressions of  $\gamma \equiv -d_\theta^-/m$  and  $m \equiv d_\theta^- + |d_\theta^+|$  permit to normalise the boundary conditions as well as to rotate the coordinates such that

$$\left.\frac{d\Theta}{d\xi}\right|_{\xi \rightarrow -\infty} = \left.\frac{d\Phi_O}{d\xi}\right|_{\xi \rightarrow -\infty} = \left.\frac{d\Theta}{d\xi}\right|_{\xi \rightarrow \infty} - 1 = \left.\frac{d\Phi_O}{d\xi}\right|_{\xi \rightarrow \infty} - 1 = 0 \quad (34)$$

Imposing  $p = \ln(Da)$  and combining Eqs. (33) and applying the boundary condition Eq. (34), the flame structure description is given by

$$\frac{d^2\Phi_O}{d\xi^2} = \frac{\Phi_O^{n_1}}{\Gamma(1+n_1)} \exp[-(\Phi_O + \gamma\xi)] \quad (35)$$

From Eq. (34), it is seen that, the value of  $\eta = \lim_{\xi \rightarrow \infty} (\Phi_O - \xi) = p/\gamma$  changes with the Damköhler number  $Da$ . From the definition of  $p$ ,

$$\exp(\eta \gamma) = \Delta \left(\frac{x_p/a}{x_f/a}\right)^4 \left(\frac{a}{\beta}\right)^2 \left(\frac{\theta_p}{\theta_f}\right)^{2+n+n_1-n_2} \frac{y_{Fp}^{n_2}}{(1-y_{Fp})^2} \exp\left(-\frac{\theta_a}{\theta_p} + \frac{\theta_a}{\theta_f}\right) \quad (36)$$

For the case  $n_1 = n_2 = 1$ , there is an approximate expression for  $\eta \gamma$  as a function of  $\gamma = -(\theta_p + l - \theta_B)/q$  (15),

$$\eta \gamma = -1.344\gamma - 4\gamma^2(1+\gamma)/(1+2\gamma) - 3\gamma^3 - \ln(1-4\gamma^2) \quad \text{for } -0.5 < \gamma < 0 \quad (37)$$

## 3. Results

An asymptotical model for droplet combustion with fuel leakage is applied to n-heptane, which is used in experimental, numerical and analytical works to simulate the gasoline. The n-heptane properties employed in this work are: ambient temperature  $T_\infty = 300K$ , boiling temperature  $T_B = 371K$ , latent heat  $L = 7645cal/mol$ , the exponent  $n = 0.5$ , n-heptane Lewis number  $L_F = 1.7$ , oxygen Lewis number  $L_O = 1.1$ , activation energy  $E_a = 30kcal/mol$ , heat of combustion  $Q = 1160kcal/mol$ , stoichiometric massic coefficient  $s = 3.52$ , and ambient oxygen mass fraction  $Y_O = 0.21$ .

Solving the Eq. (36) with Eqs. (22), (23), (24), (25) and (37), the flame temperature  $\theta_p$  is determined as a function of the modified Damköhler number  $\Delta$ . As large  $\Delta$  is as close the flame temperature  $\theta_p$  is to value of Burke-Schuman flame temperature (without leakage)  $\theta_f$ . This is so because of the reduction in the fuel leaking  $y_{Fp}$ , consequence of the increase on the reaction rate, as  $\Delta$  increases. The plot depicted in Fig. (1) confirms that dependence of the flame temperature  $\theta_p$  on  $\Delta$ . For  $\Delta = 10^3$  and  $10^4$ , the flame temperature  $\theta_p$  is very close to the Burke-Schuman flame temperature  $\theta_f = 8.7317$ , because the fuel leakage  $y_{Fp}$  is very small for these values of  $\Delta$ , as will be seen below.

Furthermore, it is also seen in Fig. (1) a large reduction on the temperature  $\theta_p$ , indicating the flame extinction. In two profiles, corresponding to  $\Delta = 10^3$ ,  $10^4$ ,  $\theta_p$  decreases fast close to the end of the droplet. These results show that

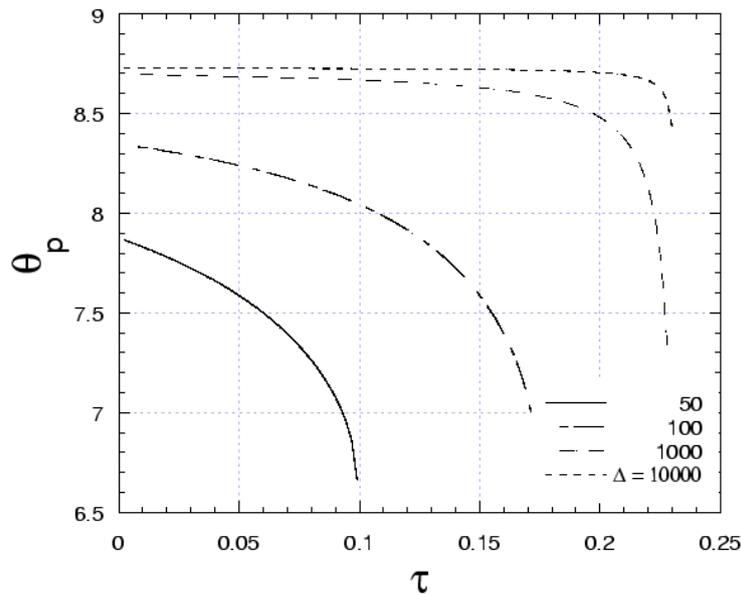


Figure 1. Variation of the flame temperature  $\theta_p$  as a function of time. The dependence of the evolution  $\theta_p$  on the Damkhöler number  $\Delta$  is presented for four representative values of  $\Delta = 50, 10, 1000, 10000$ .

the Burke-Schuman model can be applied to conditions with which  $\Delta \gg 1$ , except for a short period of time before the extinction.

For the other two profiles  $\Delta = 50, 10^2$ , the flame temperature decreases too fast even for droplet radius  $a$  of the order of unity, a  $\sim 0.8$  and  $0.6$  respectively. This behaviour for the two profiles is an indication of extinction; the fuel flow generated by the droplet vaporisation does not sustain the flame.

The variation of the flame temperature  $\theta_p$  with the Damkhöler number  $\Delta$  has a strong effect on the fuel leakage  $y_{Fp}$ , as showing in Fig. (2). Approximately, fuel leakage  $y_{Fp}$  changes accordingly to  $\sim 1/\Delta$ , for  $y_{Fp} \ll 1$ . The fast increase of the fuel leakage at the end of the stable period of time confirm the fast decrease of the flame temperature, characteristic of flame extinction.

The fuel leakage  $y_{fp}$  leads to a decrease in the flame temperature  $\theta_p$  as well as in the vaporisation rate, as showing the plot of the vaporisation constant  $\beta$  in Fig. (3).

The droplet vaporisation rate  $\lambda$  is imposed by the heat transfer from the gas to the droplet and it is determined by the combination of the flame position and the flame temperature. Therefore, to decrease the heat transfer to the droplet the flame temperature needs to decrease and the flame takes place far from the droplet. This behaviour of the flame position is seen in Fig. (4).

The variation of the vaporisation constant  $\beta$  with time leads the droplet radius square  $a^2$  not to decrease linearly with time. For  $\Delta = 10^3$  and  $10^4$ , the variation of  $\beta$  is very small in almost all vaporisation time except at the end of the droplet, then for these cases  $a^2$  follows the classical results,  $a^2 = 1 - 2\beta\tau$ , practically during the droplet life time. The discrepancy from the classical results is more evident for the other two cases,  $\Delta = 50$  and  $10^2$ , as can be seen in Fig. (5).

#### 4. Conclusion

Fuel leakage by the flame causes a reduction of the flame temperature. Consequently, the vaporisation rate decreases and the flame stabilises far from droplet. The vaporisation constant, that in the Burke-Schuman condition is a specific value and because of that the droplet radius square decreases linearly with time, under the fuel leakage condition, varies with time. The variation of the vaporisation constant with time is more evident for Damkhöler number  $\Delta < 1000$ , and under this condition the droplet radius square variation abandons the linear reduction with time.

The results show that the model considering fuel leakage by the flame can be used to simulate soot formation, but can not be used to estimate the flame position.

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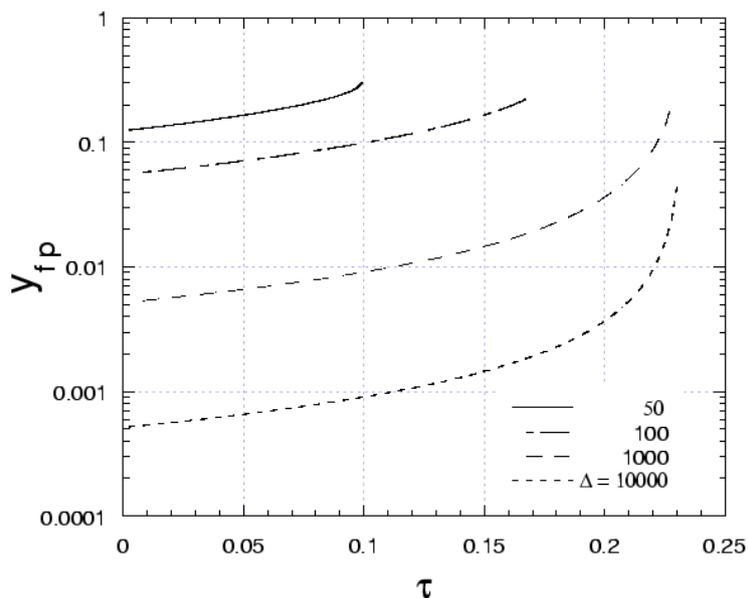


Figure 2. Variation of the fuel leakage  $y_{F_p}$  as a function of time. The dependence of the evolution  $x_p/a$  on the Damkhöler number  $\Delta$  is presented for four representative values of  $\Delta = 50, 100, 1000, 10000$

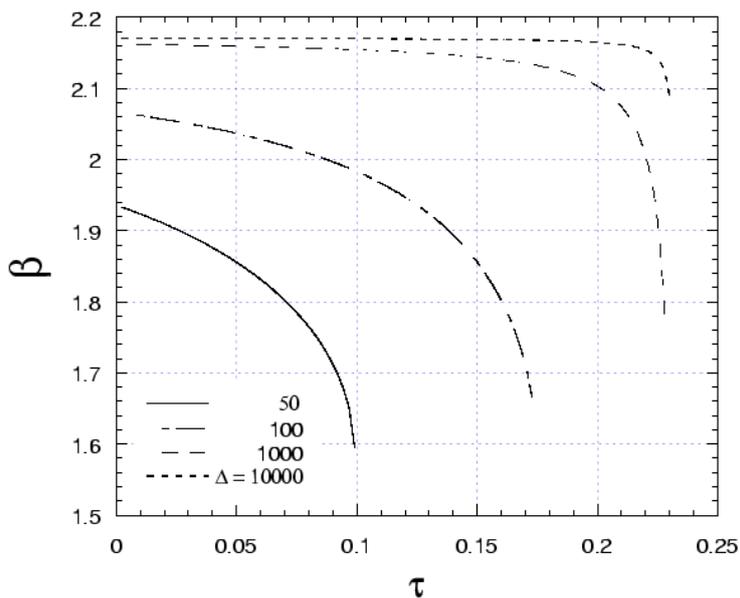


Figure 3. Variation of the vaporisation constant  $\beta$  as a function of time. The dependence of the evolution of  $\beta$  on the Damkhöler number  $\Delta$  is presented for four representative values of  $\Delta = 50, 100, 1000, 10000$ .

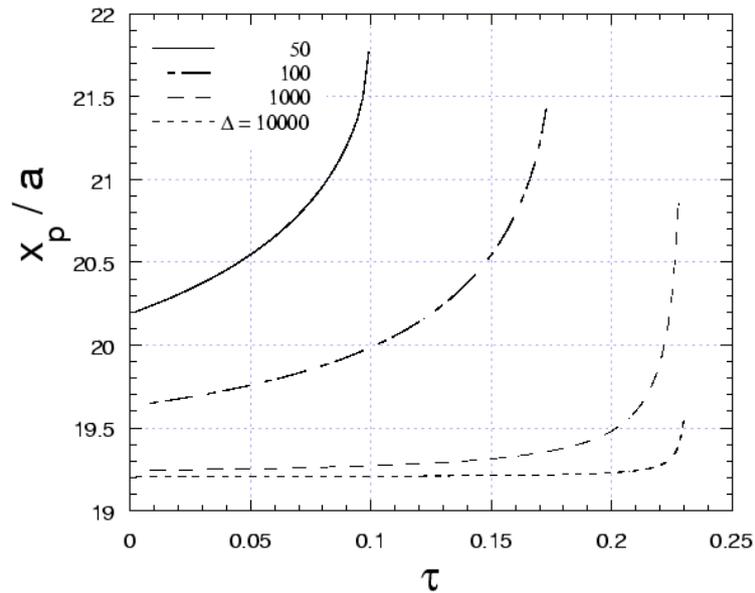


Figure 4. Variation of the stand off distance  $x_p/a$  as a function of time. The dependence of the evolution  $x_p/a$  on the Damköhler number  $\Delta$  is presented for four representative values of  $\Delta = 50, 100, 1000, 10000$ .

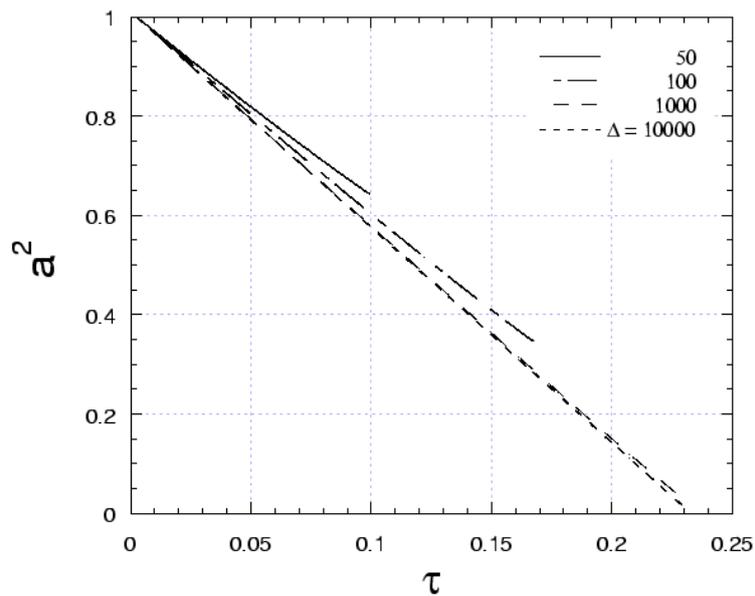


Figure 5. Variation of the square droplet radius  $a^2$  as a function of time. The dependence of the evolution  $a^2$  on the Damköhler number  $\Delta$  is presented for four representative values of  $\Delta = 50, 100, 1000, 10000$ .

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