

1. Classification <i>INPE COM.4/RPE</i> <i>C.D.U. 537.621</i>	2. Period	4. Distribution Criterion internal <input type="checkbox"/> external <input checked="" type="checkbox"/>
3. Key Words (selected by the author) <i>ITINERANT ELECTRON MAGNETISM</i> <i>SPIN WAVES</i>		
5. Report Nº <i>INPE-1594-RPE/080</i>	6. Date <i>October, 1979</i>	7. Revised by <i>R. Lawank</i> <i>Ronald Ramvand</i>
8. Title and Sub-title <i>SPIN WAVES IN ITINERANT ELECTRON</i> <i>SPIN DENSITY WAVE STATES</i>		9. Authorized by <i>Narada</i> <i>Nelson de Jesus Parada</i> <i>Director</i>
10. Sector <i>DEE</i>	Code	11. Nº of Copies <i>09</i>
12. Authorship <i>R. Kishore</i>		14. Nº of Pages <i>19</i>
13. Signature of the responsible <i>Ram Kishore</i>		15. Price
16. Summary/Notes <i>The transverse dynamical susceptibility and the spin wave stiffness constant of spin density wave (SDW) states in itinerant electron systems are studied. The Hamiltonian is transformed by using a transformation which rotates the spin coordinate system locally, so that the new-z direction lies in the direction of the local spin. For this transformed Hamiltonian, a self consistent many body theory developed by Fedro and Wilson and extended by Kishore is exploited to derive an exact expression for the transverse dynamical susceptibility. From the long-wave length limit of the spin wave spectrum, given by the poles of the transverse dynamical susceptibility, an exact formula for the spin wave stiffness constant is obtained. Within the Hartree-Fock approximation, it reduces to the Fisher's formula for antiferromagnets.</i>		
17. Remarks <i>This paper will be submitted to Physical Review B.</i>		

Spin waves in itinerant electron spin density wave states

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The transverse dynamical susceptibility and the spin wave stiffness constant of spin density wave (SDW) states in itinerant electron systems are studied. The Hamiltonian is transformed by using a transformation which rotates the spin coordinate system locally, so that the new z-direction lies in the direction of the local spin. For this transformed Hamiltonian, a self consistent many body theory developed by Fedro and Wilson and extended by Kishore is exploited to derive an exact expression for the transverse dynamical susceptibility. From the long-wave-length limit of the spin wave spectrum, given by the poles of the transverse dynamical susceptibility, an exact formula for the spin wave stiffness constant is obtained. Within the Hartree-Fock approximation, it reduces to the Fisher's formula for antiferromagnets..

I. INTRODUCTION

In 1960, Overhauser¹ showed the possibility for the existence of spin density wave (SDW) states in electron gas model within the Hartree-Fock approximation. And later on Penn², Alexander and Horwitz³, and Morris and Cornwell⁴ showed their existence in narrow band solids, using the Hubbard model within the Hartree-Fock approximation. Experimentally, SDW states have been found in transition and rare earth metals⁵. In spite of the experimental and theoretical indications for the existence of SDW states, very little is known⁶ about their collective excitations namely spin waves. In ferromagnets^{5,7}, it is now well established that in long wave-length limit, the spin wave spectrum is given as $\omega \sim D_f q^2$, where ω is the energy of the spin wave, q is the magnitude of the spin wave wave-vector, and D_f is the spin wave stiffness constant. Edwards and Fisher⁸, Corrias and Pasquale⁹, and Kishore¹⁰, by using different approaches, obtained exact microscopic formulas for D_f , from the exact expressions for the transverse dynamical susceptibility. In antiferromagnets Fisher¹¹, by using an approach similar to that of Edwards and Fisher⁸ for ferromagnets, obtained an exact formula for the stiffness constant D_a , given by the spin wave spectrum $\omega \sim D_a q$. The disadvantage of this approach, as pointed out by Fisher¹¹ himself, is that one cannot work out an exact expression for the transverse dynamical susceptibility. In this paper, we show that, by using an approach similar to that of Kishore¹⁰ for ferromagnets, it is possible to obtain exact expressions for the transverse dynamical susceptibility, as well as the spin wave stiffness constant for SDW states of which antiferromagnetic state is a special case.

We describe our itinerant electron system by the Hamiltonian

$$H = \sum_{ij\sigma} T_{ij} a_{i\sigma}^{\dagger} a_{j\sigma} + H_{e| - e|} \quad (1)$$

where T_{ij} is the transfer integral between the lattice site R_i^{\rightarrow} and R_j^{\rightarrow} ; $a_{i\sigma}^{\dagger}$ ($a_{i\sigma}$) are the creation (annihilation) operators of the electrons at the site R_i^{\rightarrow} and spin σ ; and $H_{e| - e|}$ is the Coulomb interaction between the electrons. In absence of spin-orbit interaction, which we assume, it is invariant to local spin rotations. For our formal calculation, we do not need the explicit form of $H_{e| - e|}$. But for actual calculation, one has to chose a particular form for it.

The coordinate transformation corresponding to local spin rotation, can be considered as follows. Let θ_i, ϕ_i be the polar and azimuthal angles of the direction of local spin at the site R_i^{\rightarrow} in the laboratory spherical coordinate system, then under a transformation, which rotates the spin-coordinate system locally so that the new z direction lies in the direction of the local spin, the annihilation operator $a_{i\sigma}$ in the laboratory frame is transformed into the corresponding annihilation operator $b_{i\sigma}$, in the locally transformed frame according to

$$a_{i\sigma} = e^{-i\sigma \frac{\phi_i}{2}} \cos\left(\frac{\theta_i}{2}\right) b_{i\sigma} - \sigma e^{-i\sigma \frac{\phi_i}{2}} \sin\left(\frac{\theta_i}{2}\right) b_{i-\sigma} \quad (2)$$

For SDW states ϕ_i is independent of the site index i and $\theta_i = \vec{Q} \cdot R_i^{\rightarrow}$, where \vec{Q} is the wave-vector of the SDW state. Ferro and antiferromagnetic

states correspond to $\vec{Q} = 0$ and $\vec{Q} = \vec{K}$ (half of the minimum reciprocal lattice vector) respectively

The new operator, $b_{i\sigma}$, satisfy the same commutation relations as $a_{i\sigma}$

$$\left[b_{i\sigma}^{\dagger}, b_{j\sigma'} \right]_{+} = \delta_{ij} \delta_{\sigma\sigma'} \quad (3)$$

and the Hamiltonian (1), in terms of these new operators, takes the form

$$H = \sum_{ij\sigma} T_{ij}^{\dagger} b_{i\sigma}^{\dagger} b_{j\sigma} + i \sum_{ij\sigma} T_{ij}^{-} b_{i\sigma}^{\dagger} b_{j-\sigma} + H'_{e1-e1} \quad (4)$$

where

$$T_{ij}^{\dagger} = \frac{1}{2N} \sum_{\vec{k}} T_{\vec{k}} e^{i\vec{k} \cdot (\vec{R}_i - \vec{R}_j)} \quad (5)$$

with

$$T_{\vec{k}}^{\pm} = \frac{1}{2} \left(T_{\vec{k}} - \frac{\vec{Q}}{2} \pm T_{\vec{k}} - \frac{\vec{Q}}{2} \right) \quad (6)$$

Here, N is the total number of lattice sites and H'_{e1-e1} is of the same form as H_{e1-e1} , except that operator a 's are replaced by the operator b 's.

In Sec.II, starting from the Kubo formula¹², we derive an exact expression for the transverse dynamical susceptibility by using

an approach similar to that of Kishore¹⁰ for ferromagnets. Finally, in Sec. III, from the long wave-length limit of the spin wave spectrum, given by the poles of the transverse dynamical susceptibility, an exact formula for the spin wave stiffness constant is derived. By expressing this formula in terms of two single particle self-energies, it is found that Fisher's expression¹¹ for an antiferromagnet is just a special case of our general result.

II. THE TRANSVERSE DYNAMICAL SUSCEPTIBILITY

The calculation of the transverse dynamical susceptibility starts from the Kubo formula¹² for the generalized dynamical susceptibility.

$$\chi^{\sigma\sigma'}(\vec{q}, \omega) = -g^2\mu_B^2 \sum_{ij} \chi_{ij}^{\sigma\sigma'}(\omega) e^{-i\vec{q}\cdot(\vec{R}_i - \vec{R}_j)} \quad (7)$$

where $\chi_{ij}^{\sigma\sigma'}(\omega)$ is the Fourier transform

$$\chi_{ij}^{\sigma\sigma'}(\omega) = \int_{-\infty}^{\infty} dt \chi_{ij}^{\sigma\sigma'}(t) e^{-i\omega t} \quad (8)$$

and $\chi_{ij}^{\sigma\sigma'}(t)$ is the double time retarded Green's function¹⁴

$$\chi_{ij}^{\sigma\sigma'}(t) = i\theta(t) \left\langle \left[S_i^\sigma, S_j^{\sigma'}(t) \right] \right\rangle \quad (9)$$

with

$$S_i^\sigma \equiv S_i^x + i\sigma S_i^y \equiv b_{i\sigma}^+ b_{i-\sigma} \quad (10)$$

$S_i^\sigma(t)$ is the Heisenberg operator, $S_i^\sigma = S_i^\sigma(t=0)$, and the angular brackets $\langle -- \rangle$ denote the ensemble average. For $\sigma' = \sigma$, Eq. (7) corresponds to the transverse dynamical susceptibility.

Now, we use a self-consistent many body theory developed by Fedro and Wilson¹³ for single particle Green's function and extended by Kishore¹⁰ for many-particle Green's function. A brief description of this theory is as follows. Differentiation of Eq.(9) for $\chi_{ij}^{\sigma\sigma'}(t)$ with respect to time t gives the equation of motion

$$-i \frac{\partial}{\partial t} \chi_{ij}^{\sigma\sigma'}(t) = 2\sigma \langle S^z \rangle \delta_{ij} \delta_{\sigma\sigma'} \delta(t) + i\theta(t) \langle [S_i^\sigma, L S_j^{-\sigma'}(t)] \rangle \quad (11)$$

where, for any arbitrary operator θ , the Liouville operator L is defined as

$$LQ \equiv [H, Q] \quad (12)$$

and it is assumed that

$$\frac{1}{2} \sum_{\sigma} \sigma \langle b_{i\sigma}^+ b_{i\sigma} \rangle \equiv \langle S_i^z \rangle \equiv \langle S^z \rangle \quad (13)$$

is independent of the lattice index i . The Green's function, on the right hand side of Eq.(11), is calculated by breaking the operator $S_j^{-\sigma'}(t)$ into two parts

$$S_j^{-\sigma'}(t) = P S_j^{-\sigma'}(t) + (1 - P) S_j^{-\sigma'}(t) \quad (14)$$

where the projection operator P is chosen as

$$P = \sum_{j\sigma} P_{j\sigma} \quad (15)$$

with

$$P_{j\sigma} Q = \frac{\sigma S_j^{-\sigma} \langle [S_j^\sigma, Q]_- \rangle}{2 \langle S^Z \rangle} \quad (16)$$

On substituting the identity (14) in Eq. (11) we get

$$\begin{aligned} -i \frac{\partial}{\partial t} \chi_{ij}^{\sigma\sigma'}(t) &= 2\sigma \langle S^Z \rangle \delta_{ij} \delta_{\sigma\sigma'} \delta(t) + \sum_{1\sigma_1} \Omega_{i1}^{\sigma\sigma_1} \chi_{1j}^{\sigma_1\sigma'}(t) \\ &+ i\theta(t) \langle [S_i^\sigma, L (1-P) S_j^{-\sigma'}(t)]_- \rangle \end{aligned} \quad (17)$$

where

$$\Omega_{i1}^{\sigma\sigma_1} = \frac{\sigma_1 \langle [S_i^\sigma, L S_1^{-\sigma_1}]_- \rangle}{2 \langle S^Z \rangle} \quad (18)$$

From the solution of the equation of motion of the operator $(1-P)\theta(t) S_j^{-\sigma'}(t)$, it can be shown that^{10,13}

$$\begin{aligned} (1-P)\theta(t) S_j^{-\sigma'}(t) &= \sum_{1\sigma_1} \int_0^\infty d\tau e^{i\tau(1-P)L} (1-P)L \\ &\times (\sigma_1 S_1^{-\sigma_1} / 2 \langle S^Z \rangle) \chi_{1j}^{\sigma_1\sigma'}(t-\tau) \end{aligned} \quad (19)$$

which, after substituting in Eq.(17), gives a closed equation for the Green's function $x_{ij}^{\sigma\sigma'}(t)$

$$-i \frac{\partial}{\partial t} x_{ij}^{\sigma\sigma'}(t) = 2\sigma \langle S^Z \rangle \delta_{ij} \delta_{\sigma\sigma'} \delta(t) + \sum_{l\sigma_1} \Omega_{il}^{\sigma\sigma_1} x_{lj}^{\sigma_1\sigma'}(t) + \sum_{l\sigma_1} \int_{-\infty}^{\infty} d\tau \gamma_{il}^{\sigma\sigma_1}(\tau) x_{lj}^{\sigma_1\sigma'}(t-\tau) \quad (20)$$

where

$$\gamma_{il}^{\sigma\sigma_1}(\tau) = \frac{i\sigma_1 \theta(\tau) \langle [S_i^\sigma, L e^{i\tau(1-P)L} (1-P)L S_l^{-\sigma_1}]_- \rangle}{2 \langle S^Z \rangle} \quad (21)$$

In terms of Fourier transform, Eq. (20) can be rewritten as

$$\omega x_{ij}^{\sigma\sigma'}(\omega) = 2\sigma \langle S^Z \rangle \delta_{ij} \delta_{\sigma\sigma'} + \sum_{l\sigma_1} \Omega_{il}^{\sigma\sigma_1} x_{lj}^{\sigma_1\sigma'}(\omega) + \sum_{l\sigma_1} \gamma_{il}^{\sigma\sigma_1}(\omega) x_{lj}^{\sigma_1\sigma'}(\omega) \quad (22)$$

where $\gamma_{il}^{\sigma\sigma_1}(\omega)$ is the Fourier transform of $\gamma_{il}^{\sigma\sigma_1}(t)$, and is defined according to Eq.(8). Eq.(22) can be solved by taking its Fourier transform in momentum space, defined as

$$F_{ij}^{\sigma\sigma'}(\omega) = \frac{1}{N} \sum_{\vec{q}} F^{\sigma\sigma'}(\vec{q}, \omega) e^{i\vec{q} \cdot (\vec{R}_i - \vec{R}_j)} \quad (23)$$

where $F_{ij}^{\sigma\sigma'}(\omega)$ stands for $\gamma_{ij}^{\sigma\sigma'}(\omega)$, or $\Omega_{ij}^{\sigma\sigma'}$. The substitution of Eq.(23) in Eq. (22) gives us

$$\begin{aligned} & \{\omega - \Omega^{\sigma\sigma'}(\vec{q}) - \gamma^{\sigma\sigma'}(\vec{q}, \omega)\} \chi^{\sigma\sigma'}(\vec{q}, \omega) \\ & = -2\sigma Ng^2\mu_B^2 \langle S^Z \rangle \delta_{\sigma\sigma'} + \{\Omega^{\sigma-\sigma}(\vec{q}) + \gamma^{\sigma-\sigma}(\vec{q}, \omega)\} \chi^{-\sigma\sigma'}(\vec{q}, \omega) \end{aligned} \quad (24)$$

The solution of Eq. (24) gives an exact expression for the transverse dynamical susceptibility as

$$\chi^{\sigma\sigma'}(\vec{q}, \omega) = \frac{-\sigma g^2\mu_B^2 N \langle S^Z \rangle \{\omega - A^{-\sigma-\sigma}(\vec{q}, \omega)\}}{\{\omega - A^{\sigma\sigma}(\vec{q}, \omega)\} \{\omega - A^{-\sigma-\sigma}(\vec{q}, \omega)\} - A^{\sigma-\sigma}(\vec{q}, \omega) A^{-\sigma\sigma}(\vec{q}, \omega)} \quad (25)$$

where

$$A^{\sigma\sigma'}(\vec{q}, \omega) = \Omega^{\sigma\sigma'}(\vec{q}) + \gamma^{\sigma\sigma'}(\vec{q}, \omega) \quad (26)$$

and from Eqs. (8), (18), (21), and (23)

$$\Omega^{\sigma\sigma'}(\vec{q}) = \frac{\sigma'}{2N \langle S^Z \rangle} \left\langle \left[S_{\vec{q}}^{\sigma}, L_0 S_{-\vec{q}}^{-\sigma'} \right]_- \right\rangle \quad (27)$$

and

$$\begin{aligned} \gamma^{\sigma\sigma'}(\vec{q}, \omega) = \frac{-i\sigma'}{2N \langle S^Z \rangle} \int dt e^{-i\omega t} \theta(t) \left\langle \left[L_0 S_{\vec{q}}^{\sigma}, e^{it(1-P)L} \right. \right. \\ \left. \left. \times (1-P)L_0 S_{-\vec{q}}^{-\sigma'} \right]_- \right\rangle \end{aligned} \quad (28)$$

Here,

$$S_{\vec{q}}^{\sigma} = \sum_i e^{-i\vec{q}\vec{R}_i} S_i^{\sigma} = \sum_{\vec{k}} b_{\vec{k}+\vec{q}\sigma}^{\dagger} b_{\vec{k}-\sigma} \quad (29)$$

and the operator L_0 is defined as

$$L_0 Q \equiv [H_0, Q]_- \quad (30)$$

with

$$H_0 = H - H'_{e1-e1} \quad (31)$$

In deriving Eqs. (27) and (28), we have used the facts that

$[H'_{e1-e1}, S_i^{\sigma}]_- = 0$ because of its invariance under local spin rotations and for any arbitrary operators X and Y

$$\langle [LX, Y]_- \rangle = - \langle [X, LY]_- \rangle \quad (32)$$

which follows from the cycle invariance property of the trace implied in the ensemble average.

III. SPIN-WAVE STIFFNESS CONSTANT

The spin-wave stiffness constant is obtained from the long wave-length limit of the spin-wave spectrum which, from the poles of the transverse dynamical susceptibility (25), is given by the quadratic equation

$$\omega^2 - P(\vec{q}, \omega) \omega - T(\vec{q}, \omega) = 0 \quad (33)$$

where

$$P(\vec{q}, \omega) = \sum_{\sigma} A^{\sigma\sigma}(\vec{q}, \omega) \quad (34)$$

and

$$T(\vec{q}, \omega) = A^{\sigma-\sigma}(\vec{q}, \omega) A^{-\sigma\sigma}(\vec{q}, \omega) - A^{\sigma\sigma}(\vec{q}, \omega) A^{-\sigma-\sigma}(\vec{q}, \omega) \quad (35)$$

From Eqs. (27) and (28), it can be shown that at $\vec{q} = 0$

$$P(0, \omega) = T(0, \omega) = 0 \quad (36)$$

and

$$A^{\sigma\sigma'}(0, \omega) = A^{-\sigma\sigma'}(0, \omega) = -A^{-\sigma-\sigma'}(0, \omega) \quad (37)$$

And therefore, in the long wave-length limit, Taylor's expansion of

$P(\vec{q}, \omega)$ and $T(\vec{q}, \omega)$ gives the spin-wave spectrum as

$$\omega = D_A q \quad (38)$$

where q is the magnitude of the spin-wave wave-vector \vec{q} and the spin-wave stiffness constant D_A is

$$D_A = \lim_{\substack{q \rightarrow 0 \\ \omega \rightarrow 0}} \left\{ \frac{1}{q^2} T(\vec{q}, \omega) \right\}^{1/2} \quad (39)$$

which, from Eqs. (35), (36) and (37), is given as

$$D_A = - \lim_{\substack{q \rightarrow 0 \\ \omega \rightarrow 0}} \left\{ \frac{1}{2} \sum_{\sigma} A^{\sigma\sigma}(\vec{q}, \omega) \right\} \lim_{\substack{q \rightarrow 0 \\ \omega \rightarrow 0}} \left\{ \frac{1}{q} \sum_{\sigma\sigma'} A^{\sigma\sigma'}(\vec{q}, \omega) \right\}^{1/2} \quad (40)$$

It can be rewritten in a form similar to that of Fisher¹¹ (see Eq.(40) of his paper) for antiferromagnets as

$$D_A = \frac{(2ad)^{1/2}}{2N \langle S^Z \rangle} \quad (41)$$

where from Eqs. (26), (27), (28) and (32)

$$a = \lim_{\substack{q \rightarrow 0 \\ \omega \rightarrow 0}} \left\{ \frac{1}{2} \sum_{\sigma} \left\langle \left[L_0 S_{\vec{q}}^{\sigma}, S_{-\vec{q}}^{-\sigma} \right] \right\rangle \right. \\ \left. + \frac{i}{2} \int dt e^{-i\omega t} \theta(t) \sum_{\sigma} \left\langle \left[L_0 S_{\vec{q}}^{\sigma}, e^{it(1-P)L(1-P)L_0} S_{-\vec{q}}^{-\sigma} \right] \right\rangle \right\} \quad (42)$$

and

$$d = \lim_{\substack{q \rightarrow 0 \\ \omega \rightarrow 0}} \frac{1}{2q^2} \left\{ \left\langle \left[L_0 \sum_{\sigma} S_{\vec{q}}^{\sigma}, \sum_{\sigma} S_{-\vec{q}}^{\sigma} \right] \right\rangle \right. \\ \left. + i \int dt e^{-i\omega t} \theta(t) \left\langle \left[L_0 \sum_{\sigma} S_{\vec{q}}^{\sigma}, e^{it(1-P)L(1-P)L_0} \sum_{\sigma} S_{-\vec{q}}^{\sigma} \right] \right\rangle \right\} \quad (43)$$

In the long wave-length limit, the term $L_0 \sum_{\sigma} S_{\vec{q}}^{\sigma} \equiv \left[H_0, \sum_{\sigma} S_{\vec{q}}^{\sigma} \right]$ is proportional to \vec{q} and the terms, multiplied by the projection operator P, are proportional to q^2 . Therefore, up to the order q^2 , the terms multiplied by the projection operator P do not contribute, and Eqs. (42) and (43) become

$$a = \lim_{\substack{q \rightarrow 0 \\ \omega \rightarrow 0}} \left\{ \frac{1}{2} \sum_{\sigma} \left\langle \left[L_0 S_{\vec{q}}^{\sigma}, S_{-\vec{q}}^{-\sigma} \right] \right\rangle + \frac{i}{2} \int dt e^{-i\omega t} \theta(t) \sum_{\sigma} \left\langle \left[L_0 S_{\vec{q}}^{\sigma}, e^{itL} L_0 S_{-\vec{q}}^{-\sigma} \right] \right\rangle \right\} \quad (44)$$

and

$$d = \lim_{\substack{q \rightarrow 0 \\ \omega \rightarrow 0}} \frac{1}{2q^2} \left\{ \left\langle \left[L_0 \sum_{\sigma} S_{\vec{q}}^{\sigma}, \sum_{\sigma} S_{-\vec{q}}^{\sigma} \right] \right\rangle + i \int dt e^{-i\omega t} \theta(t) \left\langle \left[L_0 \sum_{\sigma} S_{\vec{q}}^{\sigma}, e^{itL} L_0 \sum_{\sigma} S_{-\vec{q}}^{\sigma} \right] \right\rangle \right\} \quad (45)$$

It should be noted that the second terms on the right hand sides of Eqs. (44) and (45) are nothing but the Fourier transform of the Green's functions $i\theta(t) \sum_{\sigma} \left\langle \left[L_0 S_{\vec{q}}^{\sigma}, L_0 S_{-\vec{q}}^{-\sigma}(t) \right] \right\rangle$ and $i\theta(t) \left\langle \left[L_0 \sum_{\sigma} S_{\vec{q}}^{\sigma}, L_0 \sum_{\sigma} S_{-\vec{q}}^{\sigma}(t) \right] \right\rangle$ respectively. Thus, the quantities a and d given by Eqs. (44) and (45), respectively, are the same as that of Fisher¹¹. The only difference is that he worked in two sublattice model of antiferromagnets, while we worked in terms of operators transformed under a transformation which rotates the spins locally. Also, our

results are more general in the sense that they are applicable to SDW states of which antiferromagnetic state is a special case.

In Eqs. (44) and (45), the first term can be calculated exactly after substituting the commutator

$$\begin{aligned} L_0 S_{\vec{q}}^{\sigma} &= [H_0, S_{\vec{q}}^{\sigma}] \\ &= \sum_{\vec{k}} (T_{\vec{k}+\vec{q}}^+ - T_{\vec{k}}^+) b_{\vec{k}+\vec{q}\sigma}^{\dagger} b_{\vec{k}-\sigma} \\ &\quad - i\sigma \sum_{\vec{k}} (T_{\vec{k}}^- + \vec{q} \cdot b_{\vec{k}}^{\dagger} + \vec{q} \cdot -\sigma b_{\vec{k}} - \sigma T_{\vec{k}}^- b_{\vec{k}} + \vec{q}\sigma b_{\vec{k}\sigma}^{\dagger}) \end{aligned} \quad (46)$$

which one gets directly from Eqs. (4) and (31). But the second terms, in general, cannot be calculated without making some kind of approximation. The difficulty arises because of the presence of the exponential e^{itL} . We calculate these terms by dividing the Liouville operator L into two parts.

$$L = \tilde{L}_0 + \tilde{L}_1 \quad (47)$$

where \tilde{L}_0 and \tilde{L}_1 are assumed to correspond to the Hamiltonians

$$\tilde{H}_0 = \sum_{\vec{k}\sigma} (T_{\vec{k}}^+ + \Sigma_{\vec{k}\sigma}^+) b_{\vec{k}\sigma}^{\dagger} b_{\vec{k}\sigma} + i \sum_{\vec{k}\sigma} (T_{\vec{k}}^- + \Sigma_{\vec{k}\sigma}^-) b_{\vec{k}\sigma}^{\dagger} b_{\vec{k}-\sigma} \quad (48)$$

and

$$\tilde{H}_1 = H - \tilde{H}_0 \quad (49)$$

respectively. The self energies $\Sigma_{\vec{k}\sigma}^{\pm}$ are supposed to arise from the Hamiltonian H'_{e1-e1} in such a way that the effect of \tilde{H}_1 is a small perturbation to \tilde{H}_0 . It is possible to diagonalize \tilde{H}_0 in the form

$$\tilde{H}_0 = \sum_{\vec{k}} E_{\vec{k}\sigma} d_{\vec{k}\sigma}^{\dagger} d_{\vec{k}\sigma} \quad (50)$$

where

$$d_{\vec{k}}^{\dagger} = \cos\left(\frac{\theta_{\vec{k}}}{2}\right) b_{\vec{k}\sigma}^{\dagger} + i \sin\left(\frac{\theta_{\vec{k}}}{2}\right) b_{\vec{k}-\sigma}^{\dagger} \quad (51)$$

$$\tan \theta_{\vec{k}} = \frac{\sum_{\sigma} (\bar{T}_{\vec{k}} + \Sigma_{\vec{k}\sigma}^{\bar{\sigma}})}{\sum_{\sigma} (T_{\vec{k}}^{\dagger} + \Sigma_{\vec{k}\sigma}^{\dagger})} \quad (52)$$

and

$$E_{\vec{k}\sigma} = \frac{1}{2} \left\{ \sum_{\sigma} (T_{\vec{k}}^{\dagger} + \Sigma_{\vec{k}\sigma}^{\dagger}) + \sigma \left[\left\{ \sum_{\sigma} (T_{\vec{k}}^{\dagger} + \Sigma_{\vec{k}\sigma}^{\dagger}) \right\}^2 + \left\{ \sum_{\sigma} (\bar{T}_{\vec{k}} + \Sigma_{\vec{k}\sigma}^{\bar{\sigma}}) \right\}^2 \right]^{1/2} \right\} \quad (53)$$

For the exponential e^{itL} , we use the operator identity

$$e^{itL} = e^{it\tilde{L}_0} + i \int_0^t d\tau e^{i\tau\tilde{L}_0} \tilde{L}_1 e^{i\tau L} \quad (54)$$

and substitute (46) for L_0 S_q^{σ} in Eqs. (44) and (45). Then, after converting the operators b_k 's into d_k 's from (51), using the equation

$$e^{it\tilde{L}_0} d_{\vec{k}_1\sigma_1}^{\dagger} d_{\vec{k}_2\sigma_2} = e^{it(E_{\vec{k}_1\sigma_1} - E_{\vec{k}_2\sigma_2})} d_{\vec{k}_1\sigma_1}^{\dagger} d_{\vec{k}_2\sigma_2} \quad (55)$$

obtained from Eq.(50), and by performing the time integrals, Eqs. (44) and (45) become

$$\begin{aligned}
 a = & \sum_{\vec{k}} T_{\vec{k}}^{-} \sin \theta_{\vec{k}}^{-} \left(1 - \frac{T_{\vec{k}}^{-} \sin^2 \theta_{\vec{k}}^{-}}{\frac{1}{2} \sum_{\sigma} (T_{\vec{k}}^{\pm} + \Sigma_{\vec{k}\sigma}^{\pm})} \right) \sum_{\sigma} \langle d_{\vec{k}\sigma}^{+} d_{\vec{k}\sigma}^{-} \rangle \\
 & - \lim_{\substack{q \rightarrow 0 \\ \omega \rightarrow 0}} \left\{ \frac{1}{2} \int_{-\infty}^{\infty} dt e^{-i\omega t} \theta(t) \sum_{\sigma} \left\langle \left[L_0 S_{\vec{q}}^{\sigma}, \int_0^t d\tau e^{i\tau \tilde{L}_0} \tilde{L}_1 e^{i\tau L_0} S_{-\vec{q}}^{-\sigma} \right] \right\rangle \right\} \quad (56)
 \end{aligned}$$

and

$$\begin{aligned}
 d = & \frac{1}{2q^2} \sum_{\vec{k}} (\vec{q} \cdot \vec{\nabla}_{\vec{k}}) T_{\vec{k}}^{\pm} \sum_{\sigma} \langle d_{\vec{k}\sigma}^{+} d_{\vec{k}\sigma}^{-} \rangle \\
 & + \frac{1}{2q^2} \sum_{\vec{k}} \left\{ (\vec{q} \cdot \vec{\nabla}_{\vec{k}})^2 T_{\vec{k}}^{-} \sin \theta_{\vec{k}}^{-} - \frac{2(\vec{q} \cdot \vec{\nabla}_{\vec{k}} T_{\vec{k}}^{\pm})^2 \cos^3 \theta_{\vec{k}}}{\Sigma_{\vec{k}+}^{\pm} - \Sigma_{\vec{k}-}^{\pm}} \right\} \\
 & \times \sum_{\sigma} \langle d_{\vec{k}\sigma}^{+} d_{\vec{k}\sigma}^{-} \rangle \\
 & - \lim_{\substack{q \rightarrow 0 \\ \omega \rightarrow 0}} \left\{ \frac{1}{2q^2} \int_{-\infty}^{\infty} dt e^{-i\omega t} \theta(t) \left\langle \left[L_0 \sum_{\sigma} S_{\vec{q}}^{\sigma}, \int_0^t d\tau e^{i\tau \tilde{L}_0} \tilde{L}_1 e^{i\tau L_0} \sum_{\sigma} S_{-\vec{q}}^{\sigma} \right] \right\rangle \right\} \quad (57)
 \end{aligned}$$

Equations (56) and (57) are the exact expressions for the quantities a and d of the spin-wave stiffness constant D_A for itinerant

electron spin density wave-states. They hold good for all one band itinerant electron Hamiltonians provided that electron-electron interaction part $H_{e|e|}$ is invariant to the local spin rotations. Usually the Hamiltonian \tilde{H}_0 can be considered a good approximation to the total Hamiltonian H , and hence the contributions from the third terms on the right hand side of Eqs. (56) and (57) are quite small and can be neglected for all practical purposes. Thus, a good estimate of spin wave stiffness constant depends on the reliability of the self energies $\Sigma_{k\sigma}^{\pm}$. It is easy to see that within the Hartree-Fock approximations ($\Sigma_{k\sigma}^{\pm} = I n_{\sigma}$, $\Sigma_{k\sigma}^{\mp} = 0$ for the Hubbard model), the expressions for a and d are identical to that of Fisher¹¹ for antiferromagnets.

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