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14. Abstract/Notes

This paper proposes a new approach to translational image registration problems, based on the theory of sequential tests of hypotheses (S.T.H.). This leads to the development of two different methods: the first one is based on the Gaussian assumption and uses the fact that the variance of the error between two images to be registered tends to be low on the registration point. The second method uses binary images derived from the original ones. The statistical model for the resulting accumulated error is a binomial distribution and the registration position is characterized by a low probability of the binary error being one. In both methods two sequences of thresholds are employed: one leading to the rejection of the point and the other one to the eventual acceptance of it. Experimental results with both methods are presented. They include registration of a Landsat image against noisy versions of it, as well as matching of different channels of the same multispectral image. Successful registration was achieved in most cases even in low signal to noise ratio conditions, with a modest amount of computational effort.

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IMAGE REGISTRATION BY SEQUENTIAL TESTS OF HYPOTHESES: GAUSSIAN AND BINOMIAL TECHNIQUES*

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ABSTRACT

This paper proposes a new approach to translational image registration problems, based on the theory of sequential tests of hypotheses (S.T.H.). This leads to the development of two different methods: the first one is based on the Gaussian assumption and uses the fact that the variance of the error between two images to be registered tends to be low on the registration point. The second method uses binary images derived from the original ones. The statistical model for the resulting accumulated error is a binomial distribution and the registration position is characterized by a low probability of the binary error being one. In both methods two sequences of thresholds are employed: one leading to the rejection of the point and the other one to the eventual acceptance of it.

Experimental results with both methods are presented. They include registration of a Landsat image against noisy versions of it, as well as matching of different channels of the same multispectral image. Successful registration was achieved in most cases even in low signal to noise ratio conditions, with a modest amount of computational effort.

1. INTRODUCTION

In remote sensing of earth resources, it is frequently necessary to characterize a scene by taking images at different times or at varying wavelengths with several types of sensors. In all these cases, the position of the sensor can vary from one situation to another, and there is a need to establish a correspondence between different images, in the sense of being able to superimpose both images in the best possible way.

If the images do not differ in scale, rotation or projection, the problem to be solved is basically a translational one. Even when more severe distortions are involved, one of the steps in geometrically correcting one image with respect to the other consists in performing a translational registration of small areas involving well determined features (ground control points).

The main techniques that have been developed so far for solving the image registration problem can be divided into two approaches: correlation techniques and sequential similarity detection (S.S.D.A.) algorithms.

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Correlation techniques have been studied extensively. In general, they face the problem of computational costs in performing the normalized correlation for every candidate position and that of the ill defined maximum at the registration point.

S.S.D.A. algorithms were introduced by Barnea and Silverman (1), and they attempt to reduce the number of necessary computations by accumulating the error between the two images and truncating it at an early stage. Hence, only at points near or at registration the computation is performed at its full precision. Two types of threshold have been introduced by those authors: a) the constant threshold, and b) the monotonically increasing threshold. In both cases, there remains the problem of appropriately selecting the threshold level.

Several authors have worked an sequential techniques for registration. Besides the work of Barnea and Silverman, one could mention the work of Vanderbrug and Rosenfeld (2) on hierarchical search in two stages, Rosenfeld (3) on window search optimization sequence, Ramapriyan (4) on sequential search at several levels, and Wong and Hall (5, 6) also an hierarchial techniques.

This paper proposes a new approach to image registration problems, based on the sequential tests of hypotheses (S.T.H.) theory. It leads to the development of two different methods: the first based on a Gaussian model for images, and the second applied to binary images.

2. SEQUENTIAL TESTS OF HYPOTHESES

The theory of sequential tests of hypotheses was developed initially by Wald (7), who introduced the sequential probability ratio test.

In order to briefly review the ideas involved in this test (the reader is referred to the original work by Wald (7), or to chapter 4 of Hancock and Wintz's book (8) for more details), let $f(x, \theta)$ be the probability density function of the random variable being observed. Wet H_0 be the hypothesis that $\theta = \theta_0$ and H_1 the hypothesis that $\theta = \theta_1$. Successive observations on x are labeled by $x_1, x_2, \dots$ and one could define the likelihood ratio

$$\Lambda(X_n) = \frac{f(x_1\ x_2\ \dots\ x_n|\theta_1)}{f(x_1\ x_2\ \dots\ x_n|\theta_0)} = \frac{f_{1n}}{f_{0n}} \ . \tag{1}$$

The sequential probabilits ratio test is defined as follows: two constants A and B (B < A) are defined. At each stage of the experiment, the ratio f_{1n}/f_{0n} is calculated.

$$\text{If} \quad B < \frac{f_{1n}}{f_{0n}} < A \ , \tag{2}$$

The experiment continues with an additional observation.

$$\text{If} \quad \frac{f_{1n}}{f_{0n}} \geq A \ , \tag{3}$$

the process finishes with the rejection of H_0 (acceptance of H_1).

$$\text{If} \quad \frac{f_{1n}}{f_{0n}} \leq B \ , \tag{4}$$

the process finishes with the acceptance of H_0. The constants A and B are chosen in such a way that the error probabilies α and β (α = probability of rejecting H_0 when it is true, and β = probabity of accepting H_0 when it is false) be specified. It can be shown (see Hancock and Wintz (7) pg. 91) that the following relations are valid

$$A = \frac{1-\beta}{\alpha}, \tag{5}$$

$$B = \frac{\beta}{1-\alpha} \tag{6}$$

3. IMAGE REGISTRATION USING THE GAUSSIAN MODEL

3.1 THE MATHEMATICAL MODEL

The problem of translational image registration may be part of a more general problem of precisely registering two images that may differ substantially from each other from the geometrical point of view. However, one can imagine that small segments of the two images, that are approximately in coincidence with each other, would present small variations in scale and orientation. This situation is simulated in this work by the addition of Gaussian noise to one or both images. This Gaussian noise will be white, with zero mean, and independent of the signals, and also independent from each other in the two images. Furthermore, it will be assumed that the two images will be normally distributed. Besides its mathematical tractability, this assumption has received considerable support in most of the literature dealing with remote sensing imagery, notably from Landsat satellites. From these assumption, it follows that the distribution of the noisy image will be also Gaussian, with mean equal to the mean of the original image and variance given by the sum of the variances of the image and the noise; that is,

$$V_I = V_S + V_N, \tag{7}$$

where V_S is the variance of the original scene and V_N is the variance of the noise.

The proposed idea is to test the variance of the difference of the two images for each reference position, which would be given by

$$V_D^{i,j} = V_{I_1} + V_{I_2} = 2\,V_S + V_{N_1} + V_{N_2} \tag{8}$$

outside the registration point, and by

$$\hat{V}_D^{i,j} = V_{N_1} + V_{N_2}$$

at the registration point, since it is assumed that the original scene is the same and only the noises associated with each image would be, in general, non-correlated and with different variances.

A sequential probability ratio test with error probabilities α and β can be performed to test the hypothesis that $\sigma = \sigma_0$ against the alternative H_1 that $\sigma = \sigma_1$.

Set $x_1, x_2, \ldots$ be successive observations of x, that represent the difference between two pixels in the two images. The joint probability density function of the samples ($x_1, x_2, \ldots x_n$) is given by

$$f_n = \frac{1}{(2\pi)^{n/2}\,\sigma^n}\, e^{\frac{-1}{2\sigma^2} \sum_{i=1}^{n} x_i^2} \tag{10}$$

The independence of the samples, which is implicit in the previous equation, is supported by a random scanning of the pixels in the window.

The probability ratio f_{1n}/f_{0n} is calculated at each stage of the experiment, and additional observations are made if the following two inequalities are satisfied

$$\frac{\beta}{1-\alpha} < \frac{f_{1n}}{f_{0n}} < \frac{1-\beta}{\alpha} \tag{11}$$

The test terminates with the acceptance of H_0 if

$$\frac{f_{1n}}{f_{0n}} \le \frac{\beta}{1-\alpha}\ , \tag{12}$$

and with the rejection of H_0 if

$$\frac{f_{1n}}{f_{0n}} \ge \frac{1-\beta}{\alpha} \tag{13}$$

by defining

$$D = \frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2} \tag{14}$$

and

$$S = \log \frac{\sigma_1^2}{\sigma_0^2}\ , \tag{15}$$

these two inequalities can be expressed by

$$\sum_{i=1}^{n} x_i^2 \le \frac{2\log \frac{\beta}{1-\alpha} + nS}{D} = A_n \tag{16}$$

and

$$\sum_{i=1}^{n} x_i^2 \ge \frac{2\log \frac{1-\beta}{\alpha} + nS}{D} = R_n\ . \tag{17}$$

A graphical representation of the three zones is presented in Fig. 1, where

$$h_0 = \frac{2\log \frac{\beta}{1-\alpha}}{D}\ , \tag{18}$$

$$h_1 = \frac{2 \log \frac{1-\beta}{\alpha}}{D} , \qquad (19)$$

and the slope of the straight lines is given by S/D.

3.2 IMPLEMENTATION OF THE GAUSSIAN ALGORITHM

In order to implement and test the algorithm, a Landsat image from the Paraiba Valley in São Paulo State, Brazil, was used, and a 512 X 512 segment was extracted and from now on refered as the "image".

From this original image two families of images were generated, by addition of white Gaussian noise with signal to noise ratio (SNR) of 10:1, 5:1, 2:1, and 1:1. Observe that the noise from one family is non-correlated with the noise from the other.

The search area was selected as a segment of 80 X 80 pixels, and the windows were taken with the size of 32 X 32 pixels. The processing time on a Burroughs B6800 computer varied between 50 and 90 seconds.

Several combinations of images from both families were tested. The images from each family were also registered with the original image, without noise. The results for the same registration point, can be observed in tables I and II where, in the case of more than one accepted point, these are described by the number of pixels that separate them from the true registration point. On the first table, the mean (which was subtracted from the image) and variance of the image were estimated over the search area; on the second table, those parameters were estimated over the whole image. The tests that used the second parameter estimation procedure always presented a substantial increase in the number of accepted points.

It can be observed that, for SNR down to 5:1, the point accepted with the least number of tests is the true registration point. In the case of greater amount of noise, the accepted points are around the true registration point, except for a few cases. An arithmetic mean on the x and y coordinates of the largest connected region led always to the true registration, with an error always not greater than one pixel (in most cases, even less than half a pixel).

Attempts were also made to register different channels from the same image. The attempts were successful between channels 4 and 5, and 6 and 7. This can be explained in term of the image histograms, that exhibit a much greater similarity between the successfully registered channels.

4. IMAGE REGISTRATION USING THE BINARY MODEL

4.1 MATHEMATICAL FORMULATION

Binary images are frequently found in practice, for example as a result of a thresholding operation. According to Muntenan (9), who evaluated the S.S.D.A. algorithm as applied to binary images, it should be expected that p (the probability of the absolute value of the binary error being one) be approximately .5 far from the registration and approach zero close to the registration position. Experimental measurements were also made, in this work, by thresholding the original and noisy images around their mean values. These results confirmed the fact that, far from registration, the value p = .5 is very reasonable, independently of the noise level, while the value of p at registration is, in general, between .1 and .3, its value being dependent upon the noise level and the variance of the original scene. If a random sampling is performed on the window, the binary error accumulated absolute value can clearly be modeled by a binomial random variable. A sequential test of hypotheses can then be formulated to search for the registration point in the following manner: the hypothesis H_0 that the value of p (average of the original Bernouilli distribution) is low (say .1, for example) is tested against the hypothesis H_1 that this value is .5.

If a sample of n independent Bernouilli random variables is accumulated, with probability p of being equal to one, the probability that a particular sequence with dn 1's be found in the n first samples is given by

$$p^{dn} (1-p)^{n-dn} . \tag{20}$$

Under H_0, the above probability will be given by

$$p_{0n} = p_0^{dn} (1-p_0)^{n-dn} . \tag{21}$$

Under H_1, the same value will be given by

$$p_{1n} = p_1^{dn} (1-p_1)^{n-dn} . \tag{22}$$

At each stage of the test, the logarithm of the likelihood ratio can be calculated as

$$S = \log \frac{p_{1n}}{p_{0n}} = dn \log \frac{p_1}{p_0} + (n-dn) \log \frac{1-p_1}{1-p_0} \tag{23}$$

The inspection continues as long as

$$\log \frac{\beta}{1-\alpha} < S < \log \frac{1-\beta}{\alpha} , \tag{24}$$

where α and β are the error probabilities defined on section 4.1.

The sequential test terminates the first time that

$$S \geq \log \frac{1-\beta}{\alpha} \tag{25}$$

with the acceptance of $p = p_1$ or it terminates when

$$S \leq \log \frac{\beta}{1-\alpha} \tag{26}$$

with the acceptance of $p = p_0$.

By calling

$$A = \log \frac{p_1}{p_0} \tag{27}$$

and

$$B = \log \frac{1-p_1}{1-p_0} \tag{28}$$

the above inequalities can be given by

$$dn \geq \frac{\log \frac{1-\beta}{\alpha}}{A-B} + n \frac{\frac{1}{B}}{A-B} = A_n \quad (29)$$

and

$$dn \leq \frac{\log \frac{\beta}{1-\alpha}}{A-B} + n \frac{\frac{1}{B}}{A-B} = R_n \quad (30)$$

respectively

Fig. 1 can also be used to visualize the acceptance zones of p_1 (misregistration), acceptance of p_0 (registration) or continuation of the test. The initial points of the straight lines are new given by

$$h_0 = \frac{\log \frac{\beta}{1-\alpha}}{A-B} \quad (31)$$

and

$$h_1 = \frac{\log \frac{1-\beta}{\alpha}}{A-B}, \quad (32)$$

and the slope of the straight lines is given by $\frac{\frac{1}{B}}{A-B}$.

4.2 IMPLEMENTATION OF THE BINOMIAL ALGORITHM

The binomial algorithm described above was implemented on the same images as the Gaussian case, in order to provide a performance comparison.

Values for p_0 between .05 and .2 led consistently to the correct registration point. Results are summarized on Tables III and IV. They show that the binomial test seems to be more robust in the sense of always accepting the correct registration point, with the fewest number of tests. Furthermore, it also allows registration between any two channels of the multispectral image. The processing time is about the same for both algorithms.

5. CONCLUSIONS

The framework of sequential tests of hypotheses (S.T.H.) provided a formal basis for defining the translational registration problem. Furthermore, once the error probabilities α and β are chosen, the task of selecting the appropriate thresholds is straightforward.

The preliminary results obtained so far with the Gaussian and binomial sequential algorithms tend to indicate that

a) the Gaussian algorithm is adequate for moderate amounts of noise; at low SNR, spatial clustering algorithms provided the correct registration by computing the means on the x and y directions; only adjacent Landsat channels seem to be able to be registered by this method.

b) The binomial test is more robust by accepting the correct registration position with the least number of tests, even in the lowest SNR conditions; since only a few points are accepted, the application of spatial clustering techniques does not seem to be as appropriate as in case a); the performance of this algorithm with different Landsat channels is markedly superior.

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SNR	ACCEPTED		NONDECIDED
10:1	0	0	
10:1	0	0	
5:1	0	0	
	+2	+1	
	+1	0	
	-1	-1	
5:1	0	0	
	-1	-1	
	-1	0	
2:1	+2	-2	
	-1	-1	
	0	0	
	0	-1	
	-7	-8	
	-3	-1	
	-1	0	
	+1	0	
	0	-4	
	+1	+1	
	+1	+2	
	-1	-2	
2:1	0	0	
	-1	0	
	0	-1	
	+1	+1	
	-1	-1	
	+1	0	
	0	+1	

SNR	ACCEPTED		NONDECIDED	
1:1	-1	-1	+1	+2
	-18	-11	-1	+1
	0	-1		
	+1	-1		
	0	0		
	-2	0		
	-1	0		
	+1	0		
	+1	+1		
	0	+1		
	0	-2		
	-1	-2		
	-2	-1		
1:1	-1	-1	+15	-12
	0	-1	+14	-13
	0	0		
	-2	-1		
	-1	0		
	0	+1		
	+1	+1		
	+1	0		
	-2	-2		

Table I. Accepted and nondecided points; Gaussian test; one of the images is noise-free; variance measured on search area; $\alpha = \beta = 10^{-5}$

SNR	ACCEPTED		NONDECIDED
10:1, 10:1	+1 0 +2 +1 -1 +2 0 0 0 +1 +1 -1 +1 -2	+16 0 +1 0 0 +3 -1 -6 -2 -2 -1 -1 +1 -2	

Table II. Accepted points; Gaussian test; images with noncorrelated noises; variance measured on global image; $\alpha = \beta = 10^{-5}$

CHANNELS	ACCEPTED	
4 and 5	0 1 -1 0	0 2 -2 1
4 and 6	0 0 0 -1	0 1 2 0
4 and 7	0 0 0	0 1 2
5 and 6	0	0
5 and 7	0	0
6 and 7	0 0 -1 -1 0 0	0 1 -2 -3 -1 2

Table IV. Accepted points; binomial test; channels from the same multispectral image; $\alpha = \beta = 10^{-5}$

SNR	ACCEPTED	
∞/10:1	0	0
∞/5:1	0	0
∞/2:1	0 -1	0 0
∞/1:1	0 +1 -1 -1 0	0 0 -2 -1 -1
∞/10:1	0	0
∞/5:1	0	0
∞/2:1	+1 0	0 0
10:1/10:1	0	0
10:1/5:1	0	0
5:1/5:1	0 -1	0 0

Table III. Accepted points; binomial test; noisy images; $\alpha = \beta = 10^{-5}$

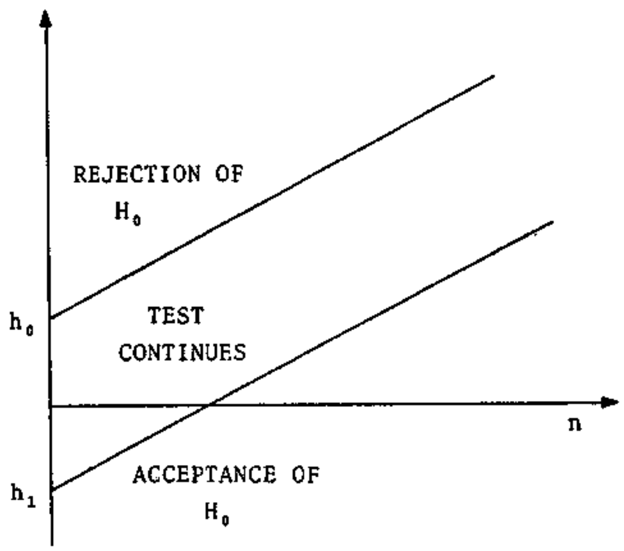

Figure 1. Regions defining the sequential probability ratio test for Gaussian and Binomial Distributions

SECOND ANNUAL WILLIAM A FISCHER MEMORIAL AWARD

Sixteenth Symposium, June 1982

PAPER #: B-14 TITLE: IMAGE REGISTRATION BY SEQUENTIAL TESTS OF HYPOTHESES: GAUSSIAN AND BINOMIAL TECHNIQUES

AUTHORS: NELSON D.A. MASCARENHAS

JOSÉ A.G. PEREIRA

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Nelson Delfino d'Ávila Mascarenhas
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May 5th, 1982
(Date)