1. Publication Nº	2. Version	3. Date	5. Distribution
INPE-3233-PRE/583		*August, 1984*	☐ Internal ☒ External ☐ Restricted
4. Origin *DTE/DPL*	Program *PRAD*		

6. Key words - selected by the author(s)
PLASMA HEATING
ELECTRON CYCLOTRON RESONANCE
NONLINEAR EFFECTS

7. U.D.C.: *533.9*

8. Title *INPE-3233-PRE/583*

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10. Nº of pages: *12*

11. Last page: *11*

12. Revised by
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13. Authorized by
Nelson de Jesus Parada
Director General

14. Abstract/Notes

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15. Remarks *To be presented in the Fourth Japan-Brazil Symposium on Science and Technology, August, 1984.*

NONLINEAR EFFECTS IN ELECTRON CYCLOTRON RESONANCE HEATING OF PLASMAS

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ABSTRACT

Mode conversion of an extraordinary wave into an upper-hybrid wave is a possible mechanism through which plasmas are heated near the electron cyclotron frequency. At the resonant layer where the mode conversion takes place the electrostatic field can attain a very large value. The ponderomotive force associated with this large amplitude upper-hybrid wave is capable of giving rise to a variety of nonlinear effects. Cavitons can be generated through nonlinear coupling of upper-hybrid envelope solitons with low-frequency lower-hybrid, electrostatic ion-cyclotron or magnetoacoustic waves.

Heating of plasmas in the electron-cyclotron frequency range appears to be an attractive method[1] as a result of recent advances in gyrotron[2,3] development and the prospect of even more powerful tunable microwave sources from free-electron lasers[4,5]. In addition to the heating of tokamak to reactor temperatures, electron-cyclotron heating can drive noninductive current to sustain a steady-state tokamak[6], create thermal barriers to improve the confinement in tandem mirror[7], and generate hot electron rings to enhance the stability of bumpy torus[8].

The heating mechanisms are mainly due to electron-cyclotron resonance or upper-hybrid resonance[1]. At low field intensities the absorption processes can be adequately handled by linear theory[1,9]. However, the field intensities generated by high-power gyrotrons and free-electron lasers are sufficient to excite parametric processes in current tokamaks[10,11], tandem mirrors[7,10] and bumpy tori[8]. Consequently, nonlinear effects may play a significant role in electron-cyclotron resonance heating of plasmas.

For a normally incident ($\vec{k} = k(x)\hat{x}$, $\vec{B}_0 = B_0(x)\hat{z}$) extraordinary electromagnetic wave in an inhomageneous plasma, the linear cold dispersion relation

$$n^2(x) = 1 - \frac{\omega_{pe}^2(x)}{\omega^2} \frac{\omega^2 - \omega_{pe}^2(x)}{\omega^2 - \omega_{uh}^2(x)} \qquad (1)$$

indicates the existence of upper-hybrid resonance ($n^2(x) \to \infty$, where n = index of refraction) when

$$\omega = \omega_{uh} \equiv (\omega_{pe}^2 + \omega_{ce}^2)^{1/2} \,, \qquad (2)$$

where $\omega_{pe}^2 = 4\pi n_0 e^2/m_e$ and $\omega_{ce} = eB_0/m_e c$. Assuming a harmonic time dependence $e^{-i\omega t}$, the linear propagation of the extraordinary wave is described by[12]

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\omega^2}{c^2} \left(1 - \frac{\omega_{pe}^2}{\omega^2} \frac{\omega^2 - \omega_{pe}^2}{\omega^2 - \omega_{uh}^2}\right) E_y = 0 , \tag{3}$$

$$E_x = - i \left(\frac{\omega_{ce}}{\omega} \frac{\omega_{pe}^2}{\omega^2 - \omega_{uh}^2}\right) E_y . \tag{4}$$

From (3) and (4) it is seen that mode conversion takes place when the wave reaches the upper-hybrid resonance layer, namely, the electromagnetic field component (E_y) vanishes, whereas the electrostatic field component (E_x) can attain a very large value. In a cold plasma $E_x \to \infty$, but in realistic situations the amplitude of mode-converted upper-hybrid waves is limited by thermal or nonlinear effects. This large amplitude electrostatic upper-hybrid wave can give rise to a variety of nonlinear effects. Under suitable conditions, the ponderomotive force associated with an intense upper-hybrid wave can induce low-frequency waves leading to parametric decay, oscillating two-stream or modulational instabilities.

Consider a large-amplitude, high-frequency, electrostatic upper-hybrid wave propagating nearly normal ($\vec{k} = k\hat{x}$) to the ambient magnetic field $\vec{B}_0 = B_0\, \hat{x}$ in a warm homogeneous plasma. The fluid equations describing the electron dynamics are

$$n_e m_e \left(\frac{\partial}{\partial t} + \vec{v}_e \cdot \nabla\right) \vec{v}_e = - e n_e \left(\vec{E} + \frac{\vec{v}_e x \vec{B}}{c}\right) - 3 v_{te}^2 \frac{\partial n_e}{\partial x} \hat{x} , \tag{5}$$

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \vec{v}_e) = 0 \ , \tag{6}$$

$$\nabla \cdot \vec{E} = 4\pi e(n_i - n_e) \ , \tag{7}$$

where $v_{Te}^2 = k_B T_e/m_e$.

Following Zakharov[13] we study the nonlinear coupling of high-frequency waves with low-frequency waves by splitting all variables into two different time-scales: the high-frequency (fast) scale follows electron motions and the low-frequency (slow) scale follows ion motions, namely,

$$\vec{E} = \vec{E}^H + \vec{E}^L \ , \tag{8}$$

$$\vec{B} = B_0 \hat{z} + \vec{B}^L \ , \tag{9}$$

$$n_e = n_0 + n_e^H + n_e^L \ , \tag{10}$$

$$n_i = n_0 + n_i^L \ . \tag{11}$$

Assuming $E_x >> E_z$ and $E_y = 0$, a substitution of (8) - (11) into (5) - (7), and linearizing in high-frequency variables yields the following wave equation for the high-frequency field[14]

$$\frac{\partial^2 E_x^H}{\partial t^2} + \omega_{uh}^2 E_x^H - D \frac{\partial^2 E_x^H}{\partial x^2} = - \frac{\omega_{pe}^2}{n_0} n_e^L E_x^H - \frac{2\omega_{ce}^2}{B_0} B_z^L E_x^H \ , \tag{12}$$

where

$$D = \frac{3\omega_{pe}^2 \; v_{Te}^2}{\omega_{pe}^2 - 3\omega_{ce}^2} \quad . \tag{13}$$

Note that the two terms on the RHS of (12) demonstrate the ponderomotive coupling of high-frequency field with low-frequency density and magnetic field oscillations.

In the absence of nonlinear coupling (n_e^L, $B_z^L \to 0$) a Fourier analysis of (12) gives the linear dispersion relation for upper-hybrid waves

$$\omega^2 = \omega_{uh}^2 + Dk^2 \; , \tag{14}$$

which shows that upper-hybrid waves have positive group dispersion ($v_g = \partial\omega/\partial k > 0$) if $\omega_{pe}^2 > 3\omega_{ce}^2$ and negative group dispersion ($v_g < 0$) if $3\omega_{ce}^2 > \omega_{pe}^2$. This forward or backward nature of upper-hybrid waves has significant implications in nonlinear wave dynamics as will be seen later.

When the amplitude of an upper-hybrid wave is sufficiently large, (13) shows that nonlinear coupling of upper-hybrid waves and low-frequency waves may appear. The nonlinear coupling is driven by the ponderomotive force associated with a large-amplitude wave originating from the fast-time averaged term $< - m_e \vec{v}_e^H \cdot \nabla \vec{v}_e^H >$ in (5), namely,

$$\vec{F}_{NL} \cong - \frac{e^2 \nabla < E^{H^2} >}{2m_e(\omega^2 - \omega_{ce}^2)} \quad . \tag{15}$$

A variety of low-frequency waves can be nonlinearly coupled to upper-hybrid waves, depending on the parameters of the system[10]. The low-frequency wave equation, in the quasi-neutral approximation ($n_i^L \cong n_e^L$), for electrostatic lower-hybrid waves ($\omega_{ci} << \omega^L << \omega_{ce}$) is

$$\left(\frac{\partial^2}{\partial t^2} - v_s^2\, \nabla^2 + \omega_{lh}^2\right) n_e^L = \frac{v_s^2}{8\pi T}\, \nabla^2 \langle E^{H^2} \rangle \ , \tag{16}$$

for electrostatic ion-cyclotron waves ($\omega^L \cong \omega_{ci}$) is

$$\left(\frac{\partial^2}{\partial t^2} - v_s^2\, \nabla^2 + \omega_{ci}^2\right) n_e^L = \frac{v_s^2}{8\pi\ T}\, \nabla^2 \langle E^{H^2} \rangle \ , \tag{17}$$

and for magnetoacoustic waves ($\omega^L << \omega_{ci}$) is

$$\left(\frac{\partial^2}{\partial t^2} - v_A^2\ (1+\beta)\nabla^2\right) B_z^L = \frac{v_A^2\ \omega^2}{2B_0\omega_{pe}^2}\, \nabla^2 \langle E^{H^2} \rangle \ , \tag{18}$$

where $\omega_{lh} = (\omega_{ce}\ \omega_{ci})^{1/2}$, $\omega_{ci} = eB_0/m_i c$, $T = \gamma_e T_e + \gamma_i T_i$, $v_s^2 = k_B\ T/m_i$, $v_A^2 = B_0^2/4\pi n_0 m_i$, and $\beta = 8\pi n_0 k_B T/B_0^2$.

Equation (12) together with (16) - (18) form three sets of nonlinear coupled wave equations describing respectively the following three-wave parametric decay instabilities

$$\begin{cases} \text{uh} \longrightarrow \text{uh (Stoke)} + \text{lh}\ , \\ \text{uh} \longrightarrow \text{uh (Stoke)} + \text{eic}\ , \\ \text{uh} \longrightarrow \text{uh (Stoke)} + \text{ma}. \end{cases} \tag{19}$$

In addition, they also describe the following four-wave modulational or oscillating two-stream (purely growing low-frequency modes) instabilities

$$
\begin{cases}
uh \longrightarrow uh\ (\text{Stoke}) + uh\ (\text{anti-Stoke}) + lh\ , \\
uh \longrightarrow uh\ (\text{Stoke}) + uh\ (\text{anti-Stoke}) + eic\ , \\
uh \longrightarrow uh\ (\text{Stoke}) + uh\ (\text{anti-Stoke}) + ma\ .
\end{cases}
\tag{20}
$$

The modulational and oscillating two-stream instabilities are of particular interest since their nonlinear development may lead to the generation of strongly nonlinear phenomena such as envelope solitons, cavitons and collapse. Introducing the modulational representation

$$E_x^H = \frac{1}{2} E(x,t) e^{-i\omega t} + \text{c.c.}\ , \tag{21}$$

where $E(x,t)$ is a slowly time-varying $(\partial E/\partial t \ll \omega E)$ complex amplitude, (13) becomes

$$i \frac{\partial E}{\partial t} + \frac{D}{2\omega} \frac{\partial^2 E}{\partial x^2} + \frac{\omega^2 - \omega_{uh}^2}{2\omega} E - \left(\frac{\omega_{pe}^2}{2\omega} \frac{n_e^L}{n_0} + \frac{\omega_{ce}^2}{\omega} \frac{B_z^L}{B_0}\right) E = 0\ . \tag{22}$$

If the process that triggers the formation of envelope solitons and cavitons occurs slowly enough, one may assume the particle density and the magnetic field perturbation by the ponderomotive force to be adiabatic. In this quasi-static limit the $\partial^2/\partial t^2$ terms in (16) - (18) can be neglected, and assuming ions weakly magnetized (i.e.,$(\omega_{lh}^2, \omega_{ci}^2) \to 0$), (16) and (17) then give

$$n_e^L \cong - \frac{\langle E^{H^2} \rangle}{8\pi T} \tag{23}$$

and (18) gives

$$B_z^L \cong - \frac{\omega^2 \langle E^{H^2} \rangle}{2\omega_{pe}^2 \ (1 + \beta) B_0} \ . \tag{24}$$

(23) and (24) describe the balance between the ponderomotive wave pressure with the plasma kinetic pressure and the magnetic field pressure, respectively. Moreover, they show that the spatial shapes of the particle density and the wave magnetic field are mirror images of the electric field energy density. Similarly, if n_e^L and B_z^L are stationary in some frame moving with the velocity V, (16) and (17) then give

$$n_e^L = \frac{\langle E^2 \rangle}{(M_s^2 - 1) \ 16\pi T} \ , \tag{25}$$

and (18) gives

$$B_z^L = \frac{\omega^2 \langle E^2 \rangle}{(M_A^2 - 1) \ 4\omega_{pe}{}^2 B_0} \ , \tag{26}$$

where $M_s = V/v_s$, $M_A = V/v_A$, $\langle E^{H^2} \rangle = \langle E^2 \rangle / 2$, and $\beta \to 0$ was assumed.

The modulational coupling of the upper-hybrid wave with either the lower-hybrid wave or the electrostatic ion-cyclotron wave can be described by the following nonlinear Schrödinger equation by substituting (25) into (22), yielding

$$i \frac{\partial E}{\partial t} + P \frac{\partial^2 E}{\partial x^2} + Q|E|^2 E = 0 \ , \tag{27}$$

with

$$P = \frac{D}{2\omega} \ , \qquad Q = \frac{\omega_{pe}^2}{(1 - M_s^2) 32\pi\omega n_0 T} \ . \tag{28}$$

Likewise, an equation of the same form as (27) with

$$P = \frac{D}{2\omega} , \qquad Q = \frac{\omega_{ce}^2 \omega}{(1 - M_A^2)\ 4\omega_{pe}^2\ B_0^2} , \tag{29}$$

which describes the coupling of the upper-hybrid wave with the magnetoacoustic wave, is obtained by substituting (26) into (22). An additional term RE (where $R = (\omega^2 - \omega_{uh}^2)/2\omega$) was removed from (27) by the transformation $E \to E \exp(iRt)$.

It is well known that the nonlinear Schrödinger equation (27) admits envelope soliton solutions if the condition $PQ > 0$ is satisfied[15]. Thus, in the positive dispersion regime (i.e., $\omega_{pe}^2 > 3\omega_{ce}^2$ and $D > 0$) the upper-hybrid envelope solitons coupled either to the lower-hybrid wave or the electrostatic ion-cyclotron wave can only exist at subsonic speeds (i.e., $M_s^2 < 1$). On the other hand, in the negative dispersion regime (i.e., $3\omega_{ce}^2 > \omega_{pe}^2$ and $D < 0$) the upper-hybrid envelope solitons coupled to the magnetoacoustic wave can only exist at super-Alfvénic speeds (i.e, $M_A{}^2 > 1$).

The intense localized field associated with upper-hybrid envelope solitons may create particle density cavitons or magnetic field cavitons, as demonstrated by (23) and (24), respectively. The cavitons can trap the solitons producing higher wave fields, which in turn create deeper cavitons. This nonlinear process continues until the wave energy is dissipated through collapse (if (12) is generalized to two or three dimensions), resulting in anamalous plasma heating or radiation.

In conclusion, it worths mentioning that recent tokamak[16] and computer simulation[17] experiments have obtained clear evidence of nonlinear effects in electron cyclotron resonance heating of plasmas. Nonlinear effects can enhance the heating efficiency[11], since in linear heating the pump energy is transferred to the electrons and, only if the confinement time is long enough, part of this energy is transferred to the ions through Coulomb collisions. However, in nonlinear heating some of the pump energy can be directly transferred to the ions through the excited low-frequency waves. The formulation presented in this paper has ignored the relativistic effects[18]. In an accurate analysis. The dynamics of nonlinear processes should be investigated by taking into account the relativistic dependence of the electron mass on the electron quiver velocity driven by the field of a large amplitude upper-hybrid wave.

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