EXPERIMENTAL STUDY OF OHMIC EFFECTS IN OPEN COAXIAL CAVITIES

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Abstract

A study of selective properties of coaxial open cylindrical resonators has been conducted experimentally and compared with theory. Mode discrimination is achieved both by exploring selective ohmic effects and examining the electrodynamic properties of the coaxial cylindrical waveguide. The effectiveness of a lossy silicon carbide coaxial insert in providing effective radial mode selection is demonstrated in that the total Q factors of TE_{mp} modes with radial index p ≥ 2 become well below the quality factors for surface TE_{m1} modes.

1 Introduction

The open cylindrical resonator finds one of its more interesting applications in cyclotron resonance masers (gyrotrons), where the open resonator consists of a weakly irregular waveguide with cutoff sections that establish the required conditions for the high-Q resonance of a given normal waveguide mode. A primary issue in the development of megawatt gyrotrons for the heating of fusion plasmas is the investigation of highly selective resonant cavities operating at very high order modes. However, some definite possibilities of an effective mode selectivity are offered by the coaxial open cylindrical resonator.

In this paper, ohmic effects in the coaxial resonator are utilized as an additional technique to weaken high-order TE modes aiming at single-mode operation of gyrotron cavities.

The resonator studied here consists of a weakly irregular waveguide into which a lossy coaxial circular cylinder was introduced (Fig. 1). The outer guide has a straight cylindrical section joined to two linear tapers and allows a higher-power electron beam to be transmitted through it. The down taper is extended far enough to ensure complete cutoff of the resonator modes whereas the up taper on the output side couples the quasi-stationary field to an outgoing travelling wave.

The presence of the lossy coaxial insert yields a cross coupling between the various TE and TM modes. However, as gyrotron cavities operates near cutoff the electron beam...
couples more strongly with TE modes than with TM ones. Therefore, throughout this work only TE\(_{mpq}\) modes will be considered.

![Schematic diagram of the coaxial resonator (dimensions in cm)](image)

Fig. 1 Schematic diagram of the coaxial resonator (dimensions in cm)

Hence, in the single mode approximation the axial distribution of the electric field satisfies the wave equation [1].

\[
d^2V(z)/dz^2 + k_{||}^2 V(z) = 0
\]  

subject to appropriate radiation conditions at the cavity ends

\[
|dV(z)/dz = i k_{||} V(z)|_{z_{in},z_{out}} = 0
\]

where \(k_{||,s} = (\omega^2/c^2 - k_{\perp,mp}^2)^{1/2}\) is the longitudinal wave number with \(s\) denoting the set of indices \((m, p, q)\); \(\omega\) is the frequency of the field and \(k_{\perp,mp}\) is the transverse wave number. The complex eigenfrequencies are close to the cutoff frequencies \(\omega_c = ck_{\perp,mp} = c\chi_{mp}/b\), where \(c\) is speed of light, \(b\) is the outer waveguide radius and \(\chi_{mp}\) denotes the \(p\)-th nontrivial root of the Bessel-Neumann combination

\[
\begin{align*}
&\left[J_m'(\chi_{mp}/C) + Z_a J_m(\chi_{mp}/C)\right] \left[N'_m(\chi_{mp} - Z_b N_m(\chi_{mp}))\right] - \\
&\left[J_m'(\chi_{mp}) - Z_b J_m(\chi_{mp})\right] \left[N'_m(\chi_{mp}/C) + Z_a N_m(\chi_{mp}/C)\right] = 0
\end{align*}
\]  

where the parameter \(C = b/a\) is defined as the ratio of the inside radius of the outer cylinder to the radius of the inner one; \(Z_{a,b} = -iZ_{a,b}/Z_0\) represents the surface impedances of the coaxial cylinders normalized to the vacuum impedance \(Z_0 = \sqrt{\mu_0/\varepsilon_0}\). The ohmic quality factor is found directly from numerical solution of eq. (3) and is expressed by

\[
Q_\Omega = \frac{Re\chi_{mp}}{2Im\chi_{mp}}
\]

Dependence of the real part of root \(\chi_{mp}\) on parameter \(C\) is shown in Fig. 2(a). Here, the external structure of the coaxial cavity is a single copper piece with an electrical conductivity \(\sigma_b = 3.8 \times 10^7 S/m\) whereas the central rod consists of a regular silicon carbide cylinder \((\sigma_a = 4.0 \times 10^3 S/m)\). The salient feature of this graph is the distinctive behavior of the curve \(Re\chi_{mp}(C)\) for two classes of modes. For surface \(TE_{m1}\) modes the quantity \(Re\chi_{mp}\) approaches \(m\) in the limit \(C \to 1\). On the other hand, that quantity increases without limit as \(C \to 1\) for volume modes with \(m \geq 2\).

The ohmic quality factor \(Q_\Omega\) for a number of modes as a function of parameter \(C\) is shown in Fig. 2(b). We note, for example, that \(Q_\Omega\) factor associated with the \(TE_{4,2}\) mode reduces from 41000 to 200 when \(C\) decreased from 10 to 1.8.
Fig. 2 Dependence of (a) Re $\chi_{mp}$ and (b) ohmic $Q$ on parameter $C$ for some TE$_{mp}$ modes (dashed lines indicated $C$-values of 3.5 and 1.8)
2 Description of Experiment

A basic schematic diagram of the experimental setup is shown in Fig. 3. Fundamental TE modes were excited [2] by means of an electric probe introduced into a 2.0 mm-diameter hole drilled through the center of the resonator's straight section. A piramidal horn antenna connected to a detector was used as a receiving device to collect partially the power reradiated by the resonator. The quality factor measured is the loaded $Q$ as determined directly from frequency reading at the half-power points on the detected spectrum.

![Schematic diagram of the experimental setup](image)

**Fig. 3** Schematic diagram of the experimental setup

3 Experimental Results

Measurements of resonant frequencies and loaded or total $Q$ factor associated with fundamental TE modes were made on two cavity configurations. In the first, we have used as the inner rod a 19.0 mm diameter cylinder, made from silicon carbide, which gives a $C$-value of 3.5; in the second case, a 37.1 mm diameter cylinder yields a $C$-value of 1.8 (Fig. 2). The observed frequency spectrum for both coaxial cavities is plotted in Fig. 4.

In particular, the coaxial resonator with $C=1.8$ (Fig. 4(b)) has less resonant modes in comparison with the $C=3.5$ design, in the operating range 9-17 GHz, as expected from Fig. 2. This last case makes it possible a more effective mode selection: modes with $p > 2$ were not excited; in addition, modes with $p=2$ have the total $Q$ below 300.

Results referring to the cavity with $C=1.8$ are summarized in Tab. 1. According to these data, the experimental resonant frequencies are in excellent agreement with those predicted by theory. The measured values of $Q_T$ are typically 10% smaller than the calculated ones.
Fig. 4 Frequency spectrum for fundamental TE modes for the (a) 3.5-design and (b) 1.8-design $C$ coaxial cavities.

4 Conclusion

Ohmic effects in coaxial open cylindrical resonators using a silicon carbide rod as the coaxial insert have been studied experimentally over the frequency range 9 to 17 GHz. Fundamental TE modes were identified through measurements of their eigenfrequencies and total or loaded $Q$ factors. Good agreement between measured and predicted resonant frequencies and total $Q$ factor was found.

We conclude that an interesting practical application offered by the coaxial cavity using silicon carbide insert is the ohmic selection in gyrotrons. The selection is due to the starting current, i.e., the minimum electron beam current required to excite a desired mode. As the starting current is proportional to $1/Q_T$, modes with low $Q_T$ require a high current to be excited.
Table 1: Measured and calculated values of resonant frequencies and Q factors of fundamental TE modes in the 1.8-design C coaxial cavity

<table>
<thead>
<tr>
<th>$\text{TE}_{m\nu} \text{mode}$</th>
<th>$F \ [\text{GHz}]$</th>
<th>$(F\pm20 \times 10^{-4}) \ [\text{GHz}]$</th>
<th>$Q_n$</th>
<th>$Q_D$</th>
<th>$Q_T$</th>
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<tr>
<td>5,1</td>
<td>8.9018</td>
<td>8.8890</td>
<td>1983</td>
<td>616</td>
<td>470</td>
<td>458 ± 10</td>
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<tr>
<td>1,2</td>
<td>10.2523</td>
<td>10.2657</td>
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<td>911</td>
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<td>6,1</td>
<td>10.5247</td>
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<td>3613</td>
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<td>727</td>
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<td>2,2</td>
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<td>1004</td>
<td>173</td>
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<td>3,2</td>
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<td>11.6853</td>
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<td>2390</td>
<td>2167 ± 55</td>
</tr>
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</table>

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References
