Application of a New Optimal Factorization of the SDRE Method in the Satellite Attitude and Orbit Control System Design with Nonlinear Dynamics

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Abstract—**The satellite Attitude and Orbit Control System (AOCS) can be designed with success by linear control theory if the satellite has slow angular motions and small attitude maneuver. However, for large and fast maneuvers, the linearized models are not able to represent all the perturbations due to the effects of the nonlinear terms present in the dynamics and in the actuators. Therefore, in such cases, it is expected that nonlinear control techniques yield better performance than the linear control techniques. One candidate technique for the design of AOCS control law under a large maneuver is the State-Dependent Riccati Equation (SDRE). SDRE entails factorization (that is, parameterization) of the nonlinear dynamics into the state vector and the product of a matrix-valued function that depends on the state itself. In doing so, SDRE brings the nonlinear system to a (not unique) linear structure having State-Dependent Coefficient (SDC) matrices and then it minimizes a nonlinear performance index having a quadratic-like structure. The non uniqueness of the SDC matrices creates extra degrees of freedom, which can be used to enhance controller performance; however, it poses challenges since not all SDC matrices fulfill the SDRE requirements. Moreover, regarding the satellite's kinematics, there is a plethora of options, e.g., Euler angles, Gibbs vector, Modified Rodrigues Parameters (MRPs), quaternions, etc. Once again, some kinematics formulations of the AOCS do not fulfill the SDRE requirements. In this paper, we evaluate the factorization options of SDC matrices for the AOCS exploring the requirements of the SDRE technique. Considering a Brazilian National Institute for Space Research (INPE) typical mission, in which the AOCS must stabilize a satellite in three-axis, the application of the SDRE technique equipped with the optimal SDC matrices can yield gains in the missions. The initial results show that MRPs for kinematics provides an optimal SDC matrix.**

Keywords-nonlinear control; SDRE method; satellite control.

I. INTRODUCTION

The design of an AOCS that involves plant uncertainties, large angle maneuvers and fast attitude control following a stringent pointing, requires nonlinear control methods to satisfy performance and robustness requirements.

An example is a typical mission of the INPE in which the AOCS must stabilize a satellite in three-axes so that the optical payload can point to the desired target with few arcsecs of pointing accuracy.

One candidate method for a nonlinear AOCS control law is the State-Dependent Riccati Equation SDRE method, originally proposed by Pearson [11] and then explored in detail by [1]-[3]. SDRE provides an effective algorithm for synthesizing nonlinear feedback control by allowing nonlinearities in the system states while offering great design flexibility through state-dependent weighting matrices. SDRE entails factorization (that is, parameterization) of the nonlinear dynamics into the state vector and the product of a matrix-valued function that depends on the state itself. In doing so, SDRE brings the nonlinear system to a (not unique) linear structure having the SDC matrices and then it minimizes a nonlinear performance index having a quadratic-like structure.

Accordingly, a suboptimal control law is carried out by a real-time solution of an Algebraic Riccati Equation (ARE) using the SDC matrices by means of a numerical algorithm. Therefore, SDRE linearizes the plant about the instantaneous point of operation and produces a constant state-space model of the system. The process is repeated in the next sampling steps, producing and controlling several state dependent linear models out of a nonlinear one.

In this paper, we evaluate the factorization options the SDC matrices for the AOCS exploring the requirements of the SDRE technique. In particular, the non uniqueness of the SDC matrices creates extra degrees of freedom, which can be used to enhance controller performance; however, it poses challenges since not all SDC matrices fulfill the SDRE requirements. Moreover, regarding the satellite's kinematics, there is a plethora of options, e.g., Euler angles, Gibbs vector, MRPs (modified Rodrigues parameters), quaternions, etc. Once again, some kinematics formulations of the AOCS do not fulfill the SDRE requirements.

A good survey of the SDRE method can be found in [1] and its systematic application to deal with a nonlinear plant in [2]. The SDRE method was applied by [4][8]-[10][14] for controlling a nonlinear system similar to the six-degree of freedom satellite model considered in this paper. In [4], a simulator using Euler angles based on commercial software

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is defined, whereas [8][9] applied quaternions on commercial software. The application of SDRE method, and, consequently, the ARE problem that arises, have already been studied in the available literature, e.g., [10] investigated the approaches for the ARE solving as well as the resource requirements for such online solving. Finally, [7][12] used SDRE method as a building block for high-level control definition, the former one using sliding model technique and the last using pursuit-evasion game. Recently, [8] proposed the usage of differential algebra to reduce the resource requirements for the real-time implementation of SDRE controllers. In fact, the intensive resource requirements for the online ARE solving is the major drawback of SDRE. Nonetheless, the SDRE method has three major advantages: (a) simplicity, (b) numerical tractability and (c) flexibility for the designer, being comparable to the flexibility in the Linear Quadratic Regulator (LQR) [9]. To the best of our knowledge, since SDC matrices are not unique, there is no work focused on the optimal arrangement of the SDC for the satellite attitude control stabilization. Such optimal arrangement has the potential to increase performance and enhance robustness. Therefore, the first contribution of the present paper is the explicit modeling of the state-space model for three-axes stabilized attitude-maneuvering satellite using quaternions, Gibbs vector and MRPs. The second, and the most important contribution, is the optimal factorization of the SDRE technique in an AOCS with nonlinear dynamics for a given set of parameters, initial condition and references for the controller. The models are evaluated for an attitude maneuver called the upside-down in the Launch and Early Orbit Phase (LEOP). In LEOP, the AOCS must dump the residual angular velocity and point the satellite solar panels towards the Sun. The results show that the MRPs provide better performance in the set of scenarios considered.

This paper is organized as follows. In Section II, the problem is described. In Section III, the satellite is modelled, and the basic equations are shared. In Section IV, the state-space models are presented and the SDRE controller is designed. In Section V, the controllability of such models is explored as well as parametrization are evaluated. In Section VI, we investigate the MRPs. In Section VII, some simulations are performed using the optimal factorization. In Section VIII, the conclusions are presented.

II. SDRE METHOD

The SDRE technique entails factorization (that is, parametrization) of the nonlinear dynamics into the state vector and the product of a matrix-valued function that depends on the state itself. In doing so, SDRE brings the nonlinear system to a (not unique) linear structure having SDC matrices given by (1).

$$\dot{\vec{x}} = A(\vec{x})\vec{x} + B(\vec{x})\vec{u}$$
$$\vec{y} = C\vec{x} \tag{1}$$

where x is the state vector and u is the control vector. Notice that the SDC form has the same structure as a linear system, but with the system matrices, A and B, being functions of the state vector. The non uniqueness of the SDC matrices creates extra degrees of freedom, which can be used to enhance controller performance, however, it poses challenges since not all SDC matrices fulfill the SDRE requirements, e.g., the pair (A,B) must be pointwise stabilizable.

The system (1) is subject of the cost functional as in (2).

$$J(\vec{x}_0,\vec{u}) = \frac{1}{2}\int_0^{\infty} (\vec{x}^T Q(\vec{x})\vec{x} + \vec{u}^T R(\vec{x})\vec{u})dt \tag{2}$$

where Q and R are the state-dependent weighting matrices. In order to ensure local stability, Q is required to be positive semi-definite for all x and R is required to be positive for all x [10]. The SDRE controller linearizes the plant about the current operating point and creates constant state space matrices so that the LQR method can be used. This process is repeated in all samplings steps, resulting in a pointwise linear model from a non-linear model, so that an ARE is solved and a control law is computed also in each step. Therefore, according to LQR theory and (1) and (2), the state-feedback control law in each sampling step is $u=-K(x)x$ and the state-dependent gain K is obtained by (3) [2].

$$K(\vec{x}) = R^{-1}(\vec{x})B^T(\vec{x})P(\vec{x}) \tag{3}$$

where P(x) is the unique, symmetric, positive-definite solution of the algebraic state-dependent Riccati equation (SDRE) given by (4).

$$P(\vec{x})A(\vec{x}) + A^T(\vec{x})P(\vec{x}) - P(\vec{x})B(\vec{x})R^{-1}(\vec{x})B^T(\vec{x})P(\vec{x}) + Q(\vec{x}) = 0 \tag{4}$$

III. SATTELITE EQUATIOS OF MOTIONS

The satellite model is designed based on a typical mission developed by INPE, in which the AOCS must stabilize the satellite in three-axis so that the optical payload can point to the desired target. Therefore, the satellite model is defined to be a three-axis stabilized, attitude-maneuvering satellite, a zero-bias-momentum system. A major control requirement is to remove the unwanted accumulated angular momentum, which would drive the satellite pointing away from the desired target. Thus, an active control system is needed to dump the residual body angular velocity that is created by perturbation torques from the space environment [6][13].

The satellite model available, which is based on the Amazonia-1 [16], uses reaction wheels (momentum exchange actuators) to provide fine attitude control and to maneuver the satellite [13]. The simulator models have two types of sensors to compute and propagate the attitude: (1) a set of Sun sensors, and (2) a gyro, which provide all the necessary information for the LEOP attitude maneuver to acquire the Sun pointing.

The satellite attitude is represented by means of quaternions in the model. Hence, the dynamic equation of

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the quaternion Q that rotates the ECI reference frame into alignment with the satellite body reference frame is as in (5) [6].

$$\dot{Q} = \frac{1}{2}\Omega(\vec{\omega})Q = \frac{1}{2}\Xi(Q)\vec{\omega}$$

$$\Omega(\vec{\omega}) \triangleq \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix}$$

$$\Xi(Q) \triangleq \begin{bmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix}, \quad (5)$$

Notice that (5) can be written as in (6), using the vector g (Gibbs vector or Rodrigues parameter) as $Q = [g^T | q_4]$ [6].

$$\dot{Q} = -\frac{1}{2}\begin{bmatrix} \omega^\times \\ \omega^T \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \frac{1}{2}q_4\begin{bmatrix} 1_{3\times3} \\ 0 \end{bmatrix}\vec{\omega} \quad (6)$$

Notice that (5) can be written as in (7) [6], using the vector p as $p = \frac{g}{1 + q_4}$.

$$\dot{p} = \frac{1}{2}[\frac{1}{2}(1 - p^T p)I_{3\times3} + p^\times + pp^T I_{3\times3}]\omega \quad (7)$$

The model of the rotational dynamics of the satellite is based on the Euler-Newton formulation and considers that the satellite has a set of 3 reaction wheels, aligned with its principal axes of inertia. One can define the inertia moment of the satellite coupled with the 3 reaction wheels by (8).

$$I_b = \vec{I} - \sum_{n=1}^{3} I_{n,s} a_n a_n^T \quad (8)$$

where Ib, I and In,s are the total, the satellite and the inertia moment of the reaction wheels in their symmetry axis an.

Assuming that there is no net external torque and using (8), the rotational dynamics of the satellite is given by (9).

$$\dot{\vec{\omega}} = (-I_b^{-1}\omega^\times I_b + I_b^{-1}(\sum_{n=1}^{3} h_{w,n}a_n)^\times)\vec{\omega} - I_b^{-1}\sum_{n=1}^{3} g_n a_n \quad (9)$$

where, ω is the angular velocity, gn is the torque generated by the n reaction wheel and hw,n is the angular momentum of the n reaction wheel about its center of mass.

IV. DESIGN OF THE SDRE CONTROLLER

The SDRE controller has to deal with two dynamics: (a) the attitude described by unit quaternions Q and (b) the angular velocity, ω, of the satellite. In (a), the attitude must be stabilized and must follow the Sun according to a given Sun vector in the satellite and in (b), the angular velocity read by the gyroscope must be as close as possible to 0. The state and the control vectors are defined by (10).

$$\begin{bmatrix} \vec{x_1} \\ \vec{x_2} \end{bmatrix} = \begin{bmatrix} Q \\ \vec{\omega} \end{bmatrix}$$

$$\begin{bmatrix} \vec{u_1} \end{bmatrix} = \begin{bmatrix} \vec{T_c} = \sum_{n=1}^{3} g_n \vec{a_n} \end{bmatrix} \quad (10)$$

Taking into account the control vector defined in (10), the state space model can be defined using (5), (9) and it is given by (11) and (12).

$$\begin{bmatrix} x_0 \\ x_2 \end{bmatrix} = \begin{bmatrix} Q \\ \omega \end{bmatrix}$$

$$\begin{bmatrix} \dot{x_0} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\Omega & 0 \\ 0 & -I_b^{-1}\omega^\times I_b + I_b^{-1}(\sum_{n=1}^{3} h_{w,n}a_n)^\times \end{bmatrix}\begin{bmatrix} x_0 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -I_b^{-1} \end{bmatrix}\begin{bmatrix} u_1 \end{bmatrix}$$

$$\begin{bmatrix} y \end{bmatrix} = 1\begin{bmatrix} x_0 \\ x_2 \end{bmatrix} \quad (11)$$

$$\begin{bmatrix} x_0 \\ x_2 \end{bmatrix} = \begin{bmatrix} Q \\ \omega \end{bmatrix}$$

$$\begin{bmatrix} \dot{x_0} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2}\Xi \\ 0 & -I_b^{-1}\omega^\times I_b + I_b^{-1}(\sum_{n=1}^{3} h_{w,n}a_n)^\times \end{bmatrix}\begin{bmatrix} x_0 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -I_b^{-1} \end{bmatrix}\begin{bmatrix} u_1 \end{bmatrix}$$

$$\begin{bmatrix} y \end{bmatrix} = 1\begin{bmatrix} x_0 \\ x_2 \end{bmatrix} \quad (12)$$

However, such state-space models, in (11) and (12), do not fulfill the SDRE requirements, in particular, the pair (A,B) is not pointwise stabilizable. Therefore, it is impossible to use such models with SDRE technique.

An alternative option for the definition of the state-space model is to use (6), which leads to (13).

$$\begin{bmatrix} x_0 \\ x_2 \end{bmatrix} = \begin{bmatrix} Q \\ \omega \end{bmatrix}$$

$$\begin{bmatrix} \dot{x_0} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}\begin{bmatrix} \omega^\times \\ \omega^T \end{bmatrix} & 0 & \begin{bmatrix} \frac{1}{2}q_4 I_{3\times3} \\ 0 \end{bmatrix} \\ 0 & 0 & -I_b^{-1}\omega^\times I_b + I_b^{-1}(\sum_{n=1}^{3} h_{w,n}a_n)^\times \end{bmatrix}\begin{bmatrix} x_0 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -I_b^{-1} \end{bmatrix}\begin{bmatrix} u_1 \end{bmatrix}$$

$$\begin{bmatrix} y \end{bmatrix} = 1\begin{bmatrix} x_0 \\ x_2 \end{bmatrix} \quad (13)$$

(13) has been shown to satisfy SDRE conditions, moreover, in it only A is a function of the state vector, consequently, that is A (x).

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Another alternative option for the definition of the state-space model is to use (7), which leads to (14).

$$\begin{bmatrix} x_3 \\ x_2 \end{bmatrix} = \begin{bmatrix} p \\ \omega \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_3 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{4}(1-p^T p)I_{3x3} + \frac{1}{2}p^{\times} + \frac{1}{2}(pp^T)I_{3x3} \\ 0 & -I_b^{-1}\omega^{\times} I_b + I_b^{-1}(\sum_{n=1}^{3} h_{w,n} a_n)^{\times} \end{bmatrix} \begin{bmatrix} x_3 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -I_b^{-1} \end{bmatrix} \begin{bmatrix} u_1 \end{bmatrix}$$

$$\begin{bmatrix} y \end{bmatrix} = I \begin{bmatrix} x_3 \\ x_2 \end{bmatrix}$$

(14)

V. INVESTIGATION OF THE CONTROLABILITY

According to [2], an effective approach for selecting the optimal state-space model for the SDRE is to attempt to maximize the pointwise stabilizability of the possible models, since pointwise control effort can be directly linked to controllability. The controllability criterion requires the value of determinant of the controllability matrix to be different from zero, therefore, a graphical comparison of the absolute value of the determinant of controllability matrix can be used to reveal when pointwise controllability is maximized. For multi-input systems, as the one studied in the present paper, the controllability matrix is not square, and then the controllability matrix multiplied by its transpose is used to evaluate the determinant.

TABLE 1. SATELLITE DATA.

Satellite characteristics	
Inertia tensor (kg.m^2)	$\begin{bmatrix} 310.0 & 1.11 & 1.01 \\ 1.11 & 360.0 & -0.35 \\ 1.01 & -0.35 & 530.7 \end{bmatrix}$
Reaction Wheels characteristics	
Inertia	0.01911
inertia tensor of 3 reaction wheels (kg.m^2)	$\begin{bmatrix} 0.01911 & 0 & 0 \\ 0 & 0.01911 & 0 \\ 0 & 0 & 0.01911 \end{bmatrix}$
Maximum torque (N.m)	0.075
Maximum angular velocity (RPM)	6000
Initial Conditions	
Attitude (degrees, XYZ)	$[0\ 0\ 180]^T$
Angular velocity (radians/second, XYZ)	$[0\ 0\ 0.024]^T$
References for the controller	
Solar vector in the body (XYZ)	$[1\ 0\ 0]^T$
Angular velocity (radians/second, XYZ)	$[0\ 0\ 0]^T$

Numerical simulations were performed to determine which of the (13) or (14) maximizes the controllability of the system for the satellite used which data are shown in Table 1. Figure 1 shows the controllability of the state-space model (SSM) defined by (13) using quaternions and the Gibbs vector, whereas Figure 2 shows the controllability of the same state-space model defined by (14).

From Figures 1 and 2, MRPs maximize the controllability through the simulation. Once MRPs provide better controllability, the next step is to further parametrize such state-model using different SDCs, which is explored in the next section below.

VI. PARAMETRIZATION OF MRP

For multivariable state-space models, as the one studied here, there are two distinct SDC matrices A1(x) and A2(x), once there is an infinite number of SDC parametrizations. Such infinite parametrizations can be constructed using (15).

$$A(x,\alpha) = \alpha A_1(x) + (1-\alpha)A_2(x) \quad (15)$$

where α is a real number and $0 \le \alpha \le 1$.

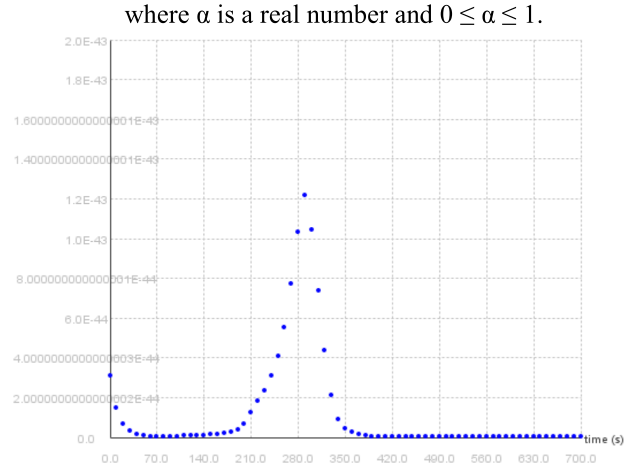

Figure 1. Controllability using quaternions and Gibbs vector.

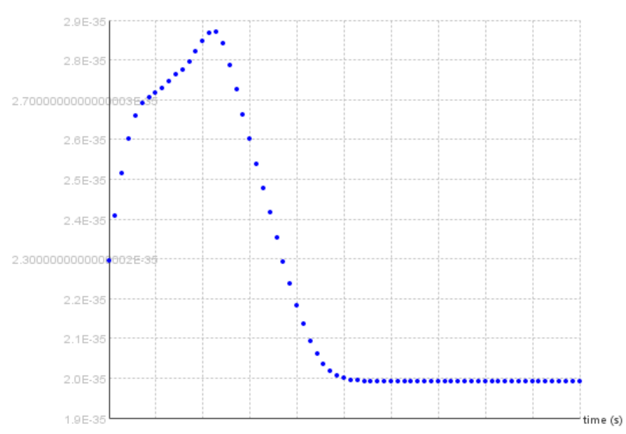

Figure 2. Controllability using MRPs.

Focusing on MRPs, which provide better controllability, and taking into account the rule of thumb that the selection of the state-dependent factorization should place a nonzero entry in the {i,j}-element of the A(x,α) matrix if the ith state derivative depends on the jth state, the state-space model is defined in (16), which is based on algebraic manipulations of (7).

$$\begin{bmatrix} x_3 \\ x_2 \end{bmatrix} = \begin{bmatrix} p \\ \omega \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_3 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}\omega^{\times} + \frac{1}{2}(\omega^T p) I_{3\times3} & \frac{1}{4}(1 - p^T p) I_{3\times3} \\ 0 & -I_b^{-1}\omega^{\times} I_b + I_b^{-1}(\sum_{n=1}^{3} h_{w,n} a_n)^{\times} \end{bmatrix} \begin{bmatrix} x_3 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -I_b^{-1} \end{bmatrix} \begin{bmatrix} u_1 \end{bmatrix}$$

$$\begin{bmatrix} y \end{bmatrix} = I \begin{bmatrix} x_3 \\ x_2 \end{bmatrix} \tag{16}$$

Figure 3 shows the controllability of the state-space model defined by (16) based on MRPs, which is better than the one shown in Figure 1 so MRPs still have better controllability than quaternions and the Gibbs vector. Nevertheless, this second option based on MRPs (16) has worse controllability than the first one defined by (14).

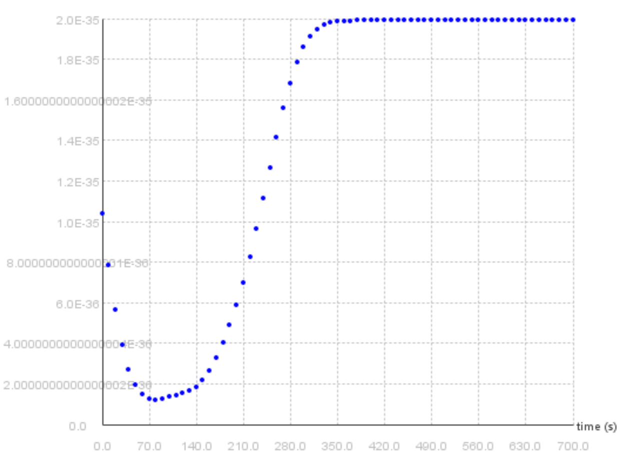

Figure 3. Controllability of MRPs of (16).

The two distinct parametrizations of (14) and (16) for the same system suggest the possibility of using (15) to evaluate a new combination of these two provides better controllability, that new parametrizations using A is given by (17).

$$A_{1_{x_3}}(x_3) = \begin{bmatrix} -\frac{1}{2}\omega^{\times} + \frac{1}{2}(\omega^T p) I_{3\times3} & \frac{1}{4}(1 - p^T p) I_{3\times3} \end{bmatrix}$$

$$A_{2_{x_3}}(x_3) = \begin{bmatrix} 0 & \frac{1}{4}(1 - p^T p) I_{3\times3} + \frac{1}{2}p^{\times} + \frac{1}{2}(pp^T) I_{3\times3} \end{bmatrix} \tag{17}$$

Applying the new parametrizations (17) into (15) and to compare the performance of resulting parametrization with different α, a simulation test was conducted with the full Monte Carlo perturbation model, in which α was randomly selected in the interval 0 ≤ α ≤ 1. The Monte Carlo model ran 90 times. Each time, the simulation used a different α with the data of Table 1. Figure 4 shows the resulting controllability of each run. It is possible to conclude that the parametrization defined by (14) is optimal since the controllability is the highest through the entire simulation. Therefore, the state-space model defined by (14), resulting in the controllability shown in Figure 2, is the optimal factorization to design the SDRE controller with nonlinear dynamics. Such conclusion is based on the characteristics of the satellite, the initial conditions and the references for the controller in Table 1. So, this is not valid for the general case and it is not valid for a different initial condition. Moreover, as (14) is defined using MRPs, which have singularity for 360o, it is neither unique nor global, whereas (13) based on quaternions and Gibbs vector is global but not unique.

VII. SIMULATION USING OPTIMAL FACTORIZATION

The satellite control using the SDRE controller designed with the optimal factorization is shown in Figures 5 and 6.

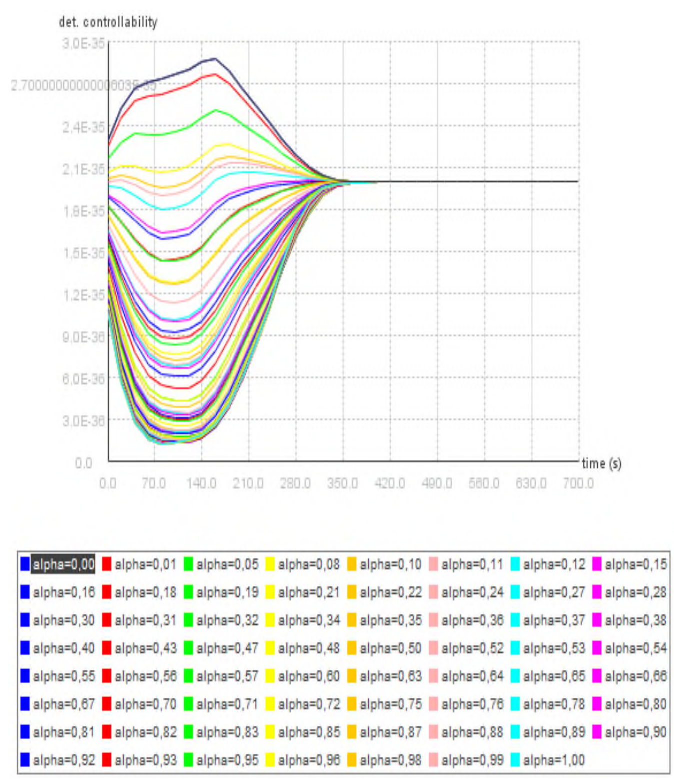

Figure 4. Controllability of MRPs using state-space model (14) and (16).

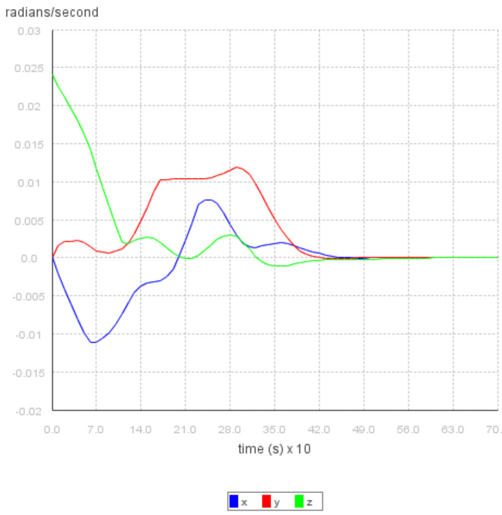

Figure 5. Control of satellite angular velocity with the optimal factorization.

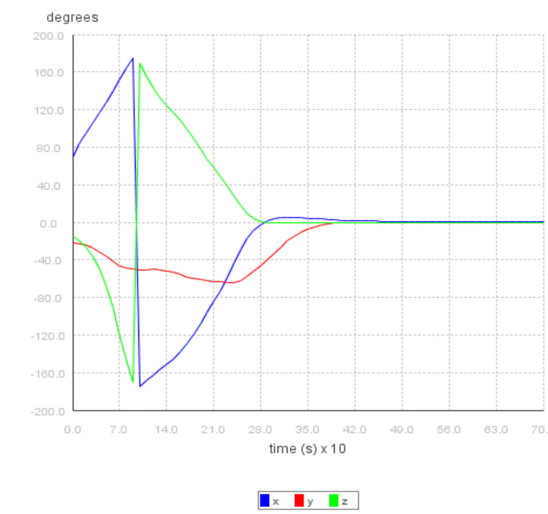

Figure 6. Control of satellite attitude with the optimal factorization.

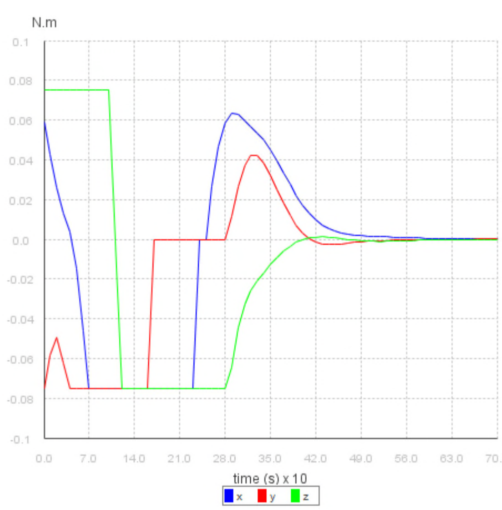

Figure 7. Reaction wheels torque with the optimal factorization.

Focusing on the actuators, Figure 7 shows that the reaction wheels torque is stabilized at about 420s.

VIII. CONCLUSION

This is a contribution for the optimal arrangement of the SDC for a three-axis stabilized satellite model. The results shown that different SDCs can produce extremely different results ranging from non-applicability of the SDRE technique to huge differences in the controllability and, consequently, in the performance and robustness of the system. Unfortunately, the optimal factorization found is neither valid for the general case nor even for a different initial condition due to the underlining nonlinear dynamics. However, the procedure applied can provide guidance for engineers. A secondary contribution is the kinematical part of the state-space models in (13), (14) and (16), since they can be used in any system that exhibits rotational motion, e.g., airplanes.

The simulations shown in the figures were performed using a portable simulator developed at INPE [5,15]; such simulator has capabilities for the unloading of the angular momentum of the reaction wheels (based on a magnetometer and a set of magnetorquers) not explored in the current paper.

One aspect discussed in this paper was whether the SDC factorization of SDRE technique in AOCS design can yield gains in the missions developed by INPE. Since performance in the LEOP is critical to the success of a mission and the simulation results show that the performance and robustness of SDRE controllers can be enhanced by optimal factorizations (in particular, with kinematics based on MRPs), then we can say that SDRE can yield gains in the missions developed by INPE. Nonetheless, its implementation requires more computing resources and tends to exhibit difficulties for verification. Therefore, it is too early to draw a definitive conclusion about the applicability of SDRE in missions at INPE. However, we can conclude that once SDRE technique is used, the optimal factorization of SDC is of utmost importance for performance and robustness of nonlinear systems controlled by such technique.

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