Cosmic Microwave Background Radiation Challenges: 2010 - 2020

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Abstract. Cosmic Microwave Background (CMB) Radiation encodes fundamental information about the early universe and is an important key to unlock a number of questions in modern cosmology. This paper highlights some important points regarding present and future observational missions and what the authors consider the most promising areas of research in CMB physics for the next decade.

Keywords: Cosmic Microwave Background (CMB); CMB polarization; CMB foregrounds; Sunyaev-Zeldovich; CMB lensing; CMB anomalies PACS:

1. INTRODUCTION

The study of the Cosmic Microwave Background Radiation is, arguably, one of the most proficuous fields in modern cosmology. Its discovery, in 1965, by Penzias and Wilson [1], was quickly followed by other astronomers who sought the confirmation of the black body behavior of its spectrum. The best determination of the CMB spectrum was made by the COBE/FIRAS instrument [2] and a recent reassessment of the CMB temperature was done by Fixsen [3]. For a review of CMB spectrum measurements, see [4]). The next quest was the search for the expected dipole emission due to our path through the CMB photons, started in the early 1970s. The best measurements were performed by [5, 6, 7]. The 1990s were extremely active, with a number of ground and balloon experiments trying to extend the COBE anisotropy results to angular scales smaller than 7°. A comprehensive list of completed, ongoing and planned CMB experiments can be found at *http://lambda.gsfc.nasa.gov/links/experimental_sites.cfm*.

The natural pursuit of CMB science started at blackbody physics and thermal equilibrium in the early universe, passing through the search for CMB anisotropies, which are the direct hint to matter fluctuations which formed the first structures in the Universe via gravitational instabilities. Scattering of CMB photons by free electrons caused primary (scalar) polarization patterns. Distortion of space-time due to primordial gravitational waves may have led to tensor polarization patterns. One pattern can be turned into the other due to lensing action between the last scattering surface (LSS) and us. Except for lensing, the CMB science above is referred to "primary" because it was produced before the decoupling, some 4×10^5 years after the beginning.

It is interesting to note the intensity of the measured signal drops from 2.7 K for blackbody radiation to 3.3×10^{-3} mK for the dipole anisotropy to $18 \times 10^{-6} \,\mu$ K for primary anisotropies to $1 \times 10^{-6} \,\mu$ K or less for polarized signals. Besides, there are various physical processes that can change the primary CMB pattern in the sky, alter its intensity or even completely mask its signal. Among them, the so-called "secondary" or "extrinsec" CMB anisotropies include the Rees-Sciama, Ostriker-Vishniac and Sunyaev-Zeldovich (SZ) effects. The amplitude of these effects is ~ hundreds of μK . For angular scales smaller than a few arcminutes, the SZ effect will be the dominant CMB contribution. For a review of SZ effect, see, e.g., [8, 9, 10]. A general discussion of CMB foregrounds can be found in [11].

The microwave signal that comes from later objects and that do not relate to CMB photons as the above processes are generaly called foreground contaminants. The primary foreground contribution comes from our Galaxy synchrotron, dust, free-free (see, e.g., [12]) and anomalous emissions [13] and from radio sources that either have a strong signal in the clearest CMB observational window (between 20 and 100 GHz) or which have few or no measurements in this window and have inverted or flat spectra. It is usually expected that radio emission in the radio and microwave window behaves as a power law. However it was shown that more than 50% of the sources detected by WMAP present either a flat or an inverted spectrum [14]. If this excess emission is not accounted for, the CMB fraction of the detected radiation will be overestimated. A large number of undetected point sources can increase the overall microwave signal collected by the antennas and cause the same effect. A very good account of foreground estimation and extraction can be found in [15, 16, 12] and references therein.

This paper intends to highlight promising points to be attacked in CMB science in the next decade, both from theoretical and observational sides. For reasons explained below, our choices were the CMB polarization, the Sunyaev-Zeldovich effect, CMB weak lensing and anomalies already observed in the CMB data.

This introduction is followed by a discussion of observational issues in section 2. The challenges we think will be faced from the theoretical point of view will be discussed in sections 3, 4 and 5, where the foregrounds for each one will also be quickly discussed. Our conclusions will be outlined in section 6.

2. OBSERVATIONAL ISSUES

Since the first COBE results, in 1992, the experimental/observational side of CMB study has undergone major leaps. Fifteen years later, there is still room for improvements from a few points of view. The COBE and WMAP space missions observed the whole sky and made anisotropy and polarization maps, as well as characterized the foreground emission with great accuracy. Planck, launched May 2009, will scrutinize the CMB anisotropies down to 5 arcmin resolution, as well as measure polarization and foregrounds in 9 frequencies, from 30 to 850 GHz. It will also produce a SZ catalog with hundreds of unknown clusters.

During these almost 2 decades, a large number of CMB experiments, both ground and balloon-based, probed smaller scales and made interferometric measurements, testing new focal plane designs, detector technologies and low-noise electronics. They also made groundbreaking discoveries, such as the flatness of the Universe and the first accurate determination of the first acoustic peak [17], the CMB anisotropy and E-mode polarization (see, e.g., [18, 19, 20, 21, 22, 23, 24]), and high signal to noise detection of the SZ effect (see, e.g., [25, 26, 27]).

We think that the two main questions to focus in the next decade are "How do we improve the current picture with Planck data" and "What Planck cannot do". It is clear that Planck will not cover angular scales smaller than a few degrees and possibly will not be able to detect the B-modes. On the other hand, it will handle the community the best CMB full sky anisotropies and polarization maps in frequencies yet unprobed [28].

The windows to follow to complement Planck science are polarization (mostly B-mode focused) and high resolution, $1 \gtrsim 2000$, measurements to target secondary anisotropies (SZ-like effects, anomalies and non-gaussian effects). The high-1 CMB science will likely be done with 5 to 10 m telescopes or interferometers. Since it requires a large sky coverage as well, it will demand either an array with a large number of detectors or a large number of interferometer units. Due to these requirements, it is highly unlikely that it will be done from balloons.

Polarization measurements raises a few issues, since the two modes come from different physics and have different observational requirements. Moreover, CMB weak lensing can change E modes into B modes. The instrumental experience on this has shown that optimizing an experiment to observe both modes and account for the transformation is not an easy task, due to the broad range of experimental requirements to cover all of them. Additionally, the galactic foregrounds (both synchrotron and dust) are heavily polarized and still poorly understood (see, e.g., accounts by the WMAP team in [16, 12]). An experiment to measure CMB polarization (specially the B-modes) must have detectors which are at least 2 orders of magnitude more sensitive than WMAP. However, detectors are presently reaching fundamental physical noise limits, and the solution for more sensitivity is either integration time or larger number of detectors in the focal plane [29]. There are already instruments trying to characterize polarized foregrounds with Brazilian participation with published results [30, 31].

From the observational point of view, foregrounds are possibly the limiting factor to cleanest CMB polarization measurements. Compact sources are the most important extragalactic foreground, both for SZ studies and for better determination of the CMB temperature power spectrum just beyond the first peak. Diffuse foregrounds (freefree, dust and synchrotron) come mostly from our Galaxy; synchrotron and dust emissions drop to a minimum, relative to CMB fluctuations, at about 70 GHz (at an angular scale of 1°). The simplest model to separate foregrounds from CMB must include synchrotron (polarized), dust (polarized), CMB (polarized), free-free(unpolarized) and anomalous dust (polarized) emissions, requiring at least 10 parameters. Since CMB polarization science (mainly the B-mode search) demands sub-microKelvin sensitivities, foreground contamination will be unavoidable and will probably be the ultimate barrier to be overcome (if ever) to determine how well CMB polarization can be measured.

The major issues, from the observational perspective, are: 1) sensitive measurement of polarized foregrounds over a wide range of frequencies, with good angular resolution; 2) realistic simulations to test both methods of foreground separation and experimental design and, 3) multifrequency measurements with enough frequencies to constrain the large (but presently unknown) number of parameters required to characterize the foregrounds.

3. COSMIC MICROWAVE BACKGROUND: PRIMARY OBSERVABLES

3.1. The spectrum

The CMB spectrum comes from an epoch when the universe was in thermodynamic equilibrium, before the decoupling era, about 380.000 years after the Big Bang. For the first year after the Big Bang, the temperature and density remained high enough for photon-creating processes (pair creation and double Compton scattering) to dominate over Hubble expansion, causing matter and radiation to remain in thermal equilibrium, characterized by the temperature. The subsequent expansion of the Universe shifted the radiation to colder temperatures but did not alter the shape of the spectrum: in the absence of later non-equilibrium interactions, the CMB will follow a blackbody spectrum.

The detection of such small signals can be significantly affected by foreground contamination from the Galaxy and extragalactic radio sources. In particular, the synchrotron component represents a significant challenge to measurements between 1 and 20 GHz. In the same frequency interval, the CMB spectrum may contain information about the epoch of reionization and subsequent structure formation, and about possible spectral distortions caused by dark matter interactions in the early universe [32].

Spectrum measurements demand sensitivities $\sim 1 \text{ mK}$ to allow the investigation of possible spectral distortions. The COBE/FIRAS result [33, 2] set the best CMB spectrum measurement, almost immediately followed by the results from Gush and colleagues ([34], obtained from an experiment flown onboard a rocket platform.

Recently, an excess in the radio emission at low frequencies (~ 3 to 10 GHz) was reported by the ARCADE team [35, 36], which is unexplained by any known physical process and is, to the moment, a route to be pursued, since positive distortions in the CMB blackbody spectrum can be caused by an unknown energy injection.

3.2. The anisotropy in the temperature distribution

Anisotropies have occupied the main scene of CMB studies since the COBE detection [7], followed by a number of small angular scales measurements and the spectacular results from WMAP [37, 16, 12]. Many references can be found in the literature about these results, including the experiments listed in website mentioned in the introduction. Since we do not think it will be a hot spot for the next decade, we will not write further about this subject.

3.3. The polarization

Since the first detection of CMB polarization by the DASI team [22], several other results followed (see, e.g., [38, 21, 39, 40, 41, 42, 18]), including the WMAP cross power spectrum TT and polarization EE spectrum (see, e.g., [43]). Planck is expected to perform the "ultimate" measurement of primary CMB temperature anisotropies. Regarding polarization, however, its goal is "only" to get the best polarization performances with the available technology at the time. [28]. The present and upcoming generation of CMB polarization experiments should shed light in important issues related to the early universe, among them: improved limits on cosmological parameters, in particular the lifting of several degeneracies that involve the optical depth through reionization; the clean signature of a stochastic background of gravitational waves generated during inflation and; evidence for weak gravitational lensing through the distortion of the CMB polarization on small scales.

The E-mode polarization and is caused by photon scattering by electrons in the last scattering surface and peaks at $l \sim 1000$ in the power spectrum. Large angle polarization comes from reionization caused by the formation of the first stars. The acoustic oscillations in the TE spectrum confirm the adiabatic origin of the fluctuations. However, the biggest prize in the CMB polarization run is the detection of the so-called B-modes, tensor perturbations probably generated by the presence of primordial gravitational waves, as predicted by the inflationary model. Table 1 summarizes the main ongoing efforts and future instruments designed to study the B-modes. For the sake of completeness, we include in this table two foreground polarization experiments that can help characterizing the synchrotron emission in frequencies below 70 GHz.

It is expected that, if the tensor-to-scalar ratio $r \approx 0.01$ or larger, the background of gravitational waves may have produced a tensor polarization intense enough to be detected, as long as we can handle the polarized foregrounds and account for the transformation from E to B modes due to weak lensing. The Planck prediction for B-mode detection is not very optimistic; however, if the reionization happened at higher redshifts ($z \simeq 14$) and the tensor to scalar ratio is r = 0.05 or larger, there is a reasonable chance (close to 50%) that Planck can detect it [29]

CMB polarization data can also be used to test alternative theories of gravitation. In these theories, vector and tensor polarization modes may leave a characteristic signature which could, in principle, be probed by measurements on the CMB E- and B-modes [44, 45]. Future polarization measurements, either from Planck or upcoming missions, can shed light on the possible presence of nontrivial gravitational wave signatures in the CMB polarization spectrum.

Experiment	Angular resolution (arcmin)	Frequency (GHz)	Goal (r)	Starting year	Platform
ABS	30	145	0.1	2010	Ground
BRAIN	~ 60	90, 150, 220	0.01	2010	Ground
Keck Array	60 - 30	100, 150, 220	0.01	2010	Ground
MBI	~ 60	90	-	2009	Ground
QUIET	28 - 12	40, 90	0.01	2008	Ground
QUIJOTE	55 - 22	11, 13, 17, 19, 30	0.05	2009	Ground
PolarBear	4 - 2.7	150, 220	0.025	2009	Ground
EBEX	8	150, 240, 410	0.02	2009	Balloon
PAPPA	30	90, 210, 300	0.01	2010	Balloon
PIPER	~ 15	200, 270, 350, 600	0.007	2013	Balloon
SPIDER	58 - 21	100, 145, 225, 275	0.01	2010	Balloon
PLANCK	33 - 5	30 - 353	0.05	2009	Satellite
C-Bass	51	5	-	2009	Ground
GEM[31]	45 - 22	5 (10)	-	2008 (2010)	Ground

TABLE 1.	Summary of	of characteristics	of B-mo	le experiments.	Adapted from	[46]	
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4. THE SUNYAEV-ZEL'DOVICH EFFECT, CMB LENSING AND ANOMALIES

The SZ effect is the most important secondary CMB anisotropy, dominant at l > 2000 and is caused by the inverse Thomson scattering by hot electrons inside galaxy clusters. One out of 100 electrons that crosses the core of a galaxy cluster is Thomson scattered, gaining energy and thus distorting the CMB blackbody spectrum, with a decrement below 218 GHz and an increment above that. Since the scattering depends only upon the electron density and temperature, the thermal SZ effect is roughly redshift independent and can be used to study the mass distribution in high redshifts. The kinetic SZ effect is a second order effect caused by the cluster drift across the line of sight. It is about an order of magnitude weaker than thermal SZ. A number of SZ measurements have been performed in the last few years, and the most promising instruments, besides Planck, are the South Pole Telescope [25], APEX [47], SZA [48] and ACT [49].

Its distinctive spectral signature can be used to independently identify high-redshift clusters, whose optical or X-ray observations are hard to detect. Additionally, it is a very powerful cosmological tool and can be used, in conjunction with observations in other wavelengths, to study the baryon mass fraction in clusters (thus estimating Ω_b indirectly), to determine H_0 (in combination with X-ray measurements), to determine clusters peculiar velocities and probe the growth of structure (and the energy density of the Universe) through number counts of clusters. The use of the SZ effect to build large galaxy cluster catalogs out to redshifts well beyond $z \gtrsim 1$ is an ongoing effort and their products are eagerly awaited. They will be used to probe dark energy and its evolution, as well as shed new light on galaxy formation processes.

The cosmic microwave background (CMB) represents a unique source for the study of gravitational lensing since, as it propagates from the last scattering surface to the observer, its geodesics can be altered by the presence of intervening matter inhomogeneities, creating lensing effects. CMB is extended across the entire sky, partially polarized, located at the extreme distance of z=1100, and is thought to have the simple, underlying statistics of a Gaussian random field. Weak lensing is caused by galaxies and, while weakly affects the CMB power spectrum, it changes the pattern of the radiation, inducing a specific form of non-Gaussianity in CMB maps. Strong lensing is produced by the distortion of CMB by the large mass concentration of galaxy clusters. The characteristic angular scales of these distortions are 1 arcmin for strong lensing and sub-arcmin for weak lensing. The lensing displacement field from a cluster can likely be modelled with good precision and used to estimate cluster masses, crucial for directly measuring N(M,z), the number density of clusters at a given mass and redshift. **This function is a highly sensitive probe of the recent growth of structure and constrains the dark energy component, as well as small neutrino masses**. A good review of CMB lensing can be found in [50].

Last, but not least, there are some unknown effects, which are generally referred to as "anomalies" in the literature and that alter the general CMB pattern. Among them we can mention the "Cold Spot" (see, e.g., [51, 52, 53, 54, 55]), North-South asymmetries (see, e.g., [56] and references therein), the low-l amplitudes (see, e.g., [57, 58] and references therein), the amplitude and orientation of CMB features (see, e.g., [59, 60, 61, 62]), anomalies in the CMB spots distribution (see, e.g., [63, 64, 59]) and general evidence of non-gaussianity in the CMB signal (see, e.g., [65, 66, 67, 68]). They can indicate departure of gaussianity, hints of a non-trivial topology and even a new physics signature.

5. REMARKS

CMB science can contribute to solve cosmological puzzles, etc. It also provides a window back to the physics of the early Universe, encoding the nature of initial density fluctuations. This paper outlines the some of the interesting features that should be attacked in the next decade, along with some actions necessary to be in the frontline of CMB research.

- 1. CMB Foregrounds (apporting data analysis expertise and instrumentation contributions)
 - design, construction and operation of detectors to measure the Galactic synchrotron foregrounds in 5 and 10 GHz (the GEM radiotelescope) to accomplish sensitive measurements of polarized foregrounds over an undersurveyed range of frequencies, with good angular resolution
 - participation in an experiment to help characterize the B-mode foreground (the COFE instrument)
 - participation in an experiment to help characterize an unknown radio foreground (the ARCADE instrument [36])
- 2. CMB Observations (apporting data analysis expertise and instrumentation contributions)
 - joining ongoing efforts of multifrequency measurements to characterize the CMB foregrounds
 - joining ongoing efforts for CMB polarization measurements
 - joining ongoing efforts for SZ and lensing measurements
 - realistic simulations to test both methods of foreground separation and experimental design
- 3. CMB Theory
 - test alternative theories of gravity, with polarization data;
 - study the origin and nature of dark matter and dark energy;
 - study the mass and structure of clusters of galaxies;
 - test the statistical isotropy hypothesis (see, e.g., [57, 59];
 - study deviations of Gaussianity in the CMB temperature fluctuations (see, e.g. [69]
 - use of CMB data as a tool to help determine the neutrino mass;
- 4. Infrastructure
 - pursue funding to keep the present instrumentation running and to be able to join future collaborations on CMB studies, contributing with subsystems built in Brazil (horns, waveguides, transitions and OMTs)
 - pursue funding for post-doctoral fellowships abroad, training researchers on data analysis and instrumentation issues and opening a door for Brazilian participation in larger CMB projects

• join efforts to create a CMB network in Brazil, stirring a more active interaction among researchers from theoretical and observational sides

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