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Closed Loop Model Reference Adaptive Control for a Satellite Launcher

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Abstract. The viability of adaptive systems has often been a source of concern, in part because of transient performance and parameter boundedness. This work aims to study a specific class of model reference adaptive control where a feedback loop is used as part of the reference model in an attempt of providing better transient response. The controller stability is proved and its performance is analyzed when applied to a satellite launcher.

Keywords. Adaptive Control, MRAC, Satellite Launcher

1 Introduction

When dealing with model reference adaptive control systems, simply verifying the controller stability may not be enough. As shown by [8], larger adaptive gains may stimulate unmodeled frequencies during its transient regiment, often leading to the plant behaving very differently than the reference model [10]. Recently, a new class of model reference adaptive controller, using a so called closed look reference model (CRM), has shown promising results during this critical phase. More specifically, this architecture has shown a reasonable reduction in the transitory oscillations, even in cases when the adaptive gains increase [1].

Initially, purposed by [4], this structure appears in other works, such as [2,9] and, with a larger focus on the system transient behavior, in [3] and [1].

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2 Methodology

The main goal of this paper is to study the performance of closed loop reference model adaptive controllers when applied to satellite launch vehicles, especially with respect to its tuning parameters.

2.1 Satellite Launch Vehicle Dynamics

As remarked by [5], satellite launcher are open loop unstable systems, whose plant dynamics are dependent on time variant parameters such as mass, moments of inertia, aerodynamics characteristics and dynamic pressure.

If thrust vector control is used, one may consider a preliminary design model for the vehicle dynamics, described by Eq. (1).

$$\frac{\theta}{\beta} = \frac{M_\beta}{s^2 - M_\alpha} \tag{1}$$

Where θ is the vehicle angular position, β is the control signal related to the nozzle deflection, M_β is the control efficiency and M_α is the aerodynamic efficiency.

The efficiency coefficients are time variant and are a function of the dynamic pressure P_q, the reference area S, the static margin l_a, the thrust force FE, the stability derivative C_{n_α}, the moment of inertia I_{yy} and the control force lever l_c. The expression for each coefficient is given by Eq. (2)

$$\begin{aligned} M_\alpha &= \frac{C_{n_\alpha} P_q S l_a}{I_{yy}} \\ M_\beta &= \frac{FE l_c}{I_{yy}} \end{aligned} \tag{2}$$

2.2 Adaptive Control Model

Consider a dynamic system in which the plant is described by Eq. (3).

$$\dot{x} = Ax + Bu + \xi(t) \tag{3}$$

In which x represents the system state, A and B are unknown matrices and it is assumed that the pair (A,B) is controllable. The function $\xi(t)$ models extern bounded disturbances, so that $\|\xi(t)\| \leq \xi_{max} > 0$, and that ξ_{max} is a known value. It is assumed that all states are accessible.

The reference model used is a closed loop reference model (CRM) proposed by [3] and described on Eq. (4).

$$\dot{x}_{ref} = A_{ref} x_{ref} + B_{ref} r + L_\nu (x - x_{ref}) \tag{4}$$

In which x_{ref} is the reference model state, A_{ref} and B_{ref} are known matrices and A_{ref} is Hurwitz. The feedback gain L_ν is calculated as $L_\nu = P_\nu R_\nu^{-1}$, where P_ν is the solution for the Ricatti equation shown in Eq. (5).

$$P_\nu A_{ref}^T + A_{ref} P_\nu - P_\nu R_\nu^{-1} P_\nu + Q_\nu = 0 \tag{5}$$

The weight matrices are selected so that $Q_\nu = Q_0 + (\nu+1)\nu^{-1} I$ and $R_\nu = \nu(\nu+1)^{-1} I$, I is the identity matrix and $\nu > 0$ is a constant parameter.

The control law selected is presented in Eq. (6). The estimated gains $\hat{K}_x$ and $\hat{K}_r$ follow the adaptation law described by Eq. (7). The term $\sigma\|e^T P_\nu^{-1} B\|$ represents a e-modification necessary for guaranteed robustness under bounded disturbances [6].

$$u = \hat{K}_x^T x + \hat{K}_r^T r \tag{6}$$

$$\begin{cases} \dot{\hat{K}}_x = -\Gamma_x [x e^T P_\nu^{-1} B + \sigma \|e^T P_\nu^{-1} B\| \hat{K}_x] \\ \dot{\hat{K}}_r = -\Gamma_r [r e^T P_\nu^{-1} B + \sigma \|e^T P_\nu^{-1} B\| \hat{K}_r] \end{cases} \tag{7}$$

Fig. 1 shows a block diagram of the control system implementation.

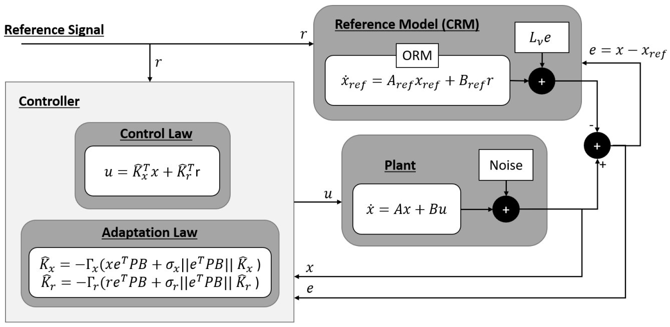

Figure 1: Block diagram of closed loop reference model (CRM) implementation.

2.2.1 Stability Analysis

The state error dynamics, in that $e = x - x_{ref}$ are described by Eq. (8).

$$\dot{e} = (A_{ref} - L_\nu) e + B[\Delta K_x^T + \Delta K_r^T] + \xi(t) \tag{8}$$

Therefore, one may write a candidate Lyapunov equation, given by Eq. (9), where $\tilde{P} = P_\nu^{-1}$.

$$V(e, \Delta K_x, \Delta K_r) = e^T \tilde{P} e + tr(\Delta K_x^T \Gamma_x^{-1} \Delta K_x + \Delta K_r^T \Gamma_r^{-1} \Delta K_r) \tag{9}$$

It can be shown that, for the control adaptation law presented in Eq. (7), the time derivative of V is shown in Eq. (10).

$$\begin{aligned} \dot{V}(e, \Delta K_x, \Delta K_r) = &- e^T \tilde{Q} e + 2 e^T \tilde{P} \xi(t) \\ &- 2tr(\Delta K_x^T \sigma \|e^T \tilde{P} B\| \hat{K}_x) - 2tr(\Delta K_r^T \sigma \|e^T \tilde{P} B\| \hat{K}_r) \end{aligned} \tag{10}$$

As [3] points out, the Lyapunov concept of stability is not designed for systems with disturbances. In this case, one must consider stability in the uniform ultimate boundedness sense. By this definition, even if a solution is not uniformly bounded initially, it may become uniform ultimately bounded in a finite amount of time [7].

To show that the solution is bounded, one must prove that $\dot{V}(e, \Delta K_x, \Delta K_r) > 0$ inside a compact set, but $\dot{V}(e, \Delta K_x, \Delta K_r) \leq 0$ outside it [7]. It is possible to rewrite Eq. (10) so that its value may be described by the inequality (11).

$$\dot{V}(e, \Delta K_x, \Delta K_r) \leq -\lambda_{min}(\tilde{Q}) \left[\|e\| - \frac{\lambda_{max}(\tilde{P})}{\lambda_{min}(\tilde{Q})} \xi_{max} \right]^2 + \frac{\lambda^2_{max}(\tilde{P})}{\lambda_{min}(\tilde{Q})} \xi^2_{max} - 2\sigma \|e^T \tilde{P} B\| \left[\left(\|\Delta K_x\|_F - \frac{\|K_x\|_F}{2} \right)^2 - \frac{\|K_x\|^2_F}{4} + \left(\|\Delta K_r\|_F - \frac{\|K_r\|_F}{2} \right)^2 - \frac{\|K_r\|^2_F}{4} \right] \tag{11}$$

Therefore, $\dot{V}(e, \Delta K_x, \Delta K_r) \leq 0$ if Eqs. (12-14) are true.

$$\|e\| \geq \frac{\lambda_{max}(\tilde{P}) \xi_{max}}{\lambda_{min}(\tilde{Q})} + \sqrt{\frac{\lambda^2_{max}(\tilde{P})}{\lambda^2_{min}(\tilde{Q})} \xi^2_{max} + \frac{\sigma \|e^T \tilde{P} B\|}{\lambda_{min}(\tilde{Q})} \left[\frac{\|K_x\|^2_F}{2} + \frac{\|K_r\|^2_F}{2} \right]} = c_1 \tag{12}$$

$$\|\Delta K_x\|_F \geq \frac{\|K_x\|_F}{2} + \frac{1}{2} \sqrt{\frac{\xi^2_{max}}{\sigma \|e^T \tilde{P} B\|} \frac{\lambda^2_{max}(\tilde{P})}{\lambda^2_{min}(\tilde{Q})} + \frac{\|K_x\|^2_F}{2} + \frac{\|K_r\|^2_F}{2}} = c_2 \tag{13}$$

$$\|\Delta K_r\|_F \geq \frac{\|K_r\|_F}{2} + \frac{1}{2} \sqrt{\frac{\xi^2_{max}}{\sigma \|e^T \tilde{P} B\|} \frac{\lambda^2_{max}(\tilde{P})}{\lambda^2_{min}(\tilde{Q})} + \frac{\|K_x\|^2_F}{2} + \frac{\|K_r\|^2_F}{2}} = c_3 \tag{14}$$

Thus, $\dot{V} < 0$ outside the compact set E_0, defined by Eq. (15).

$$E_0 = \left\{ \begin{array}{l} (\|e\|, \|\Delta K_x\|_F, \|\Delta K_r\|_F) : \quad \lambda_{min}(\tilde{Q}) \left[\|e\| - \frac{\lambda_{max}(\tilde{P})}{\lambda_{min}(\tilde{Q})} \xi_{max} \right]^2 + \\ 2\sigma \|e^T \tilde{P} B\| \left[\left(\|\Delta K_x\|_F - \frac{\|K_x\|_F}{2} \right)^2 + \left(\|\Delta K_r\|_F - \frac{\|K_r\|_F}{2} \right)^2 \right] \\ \leq \frac{\lambda^2_{max}(\tilde{P})}{\lambda_{min}(\tilde{Q})} \xi^2_{max} + 2\sigma \|e^T \tilde{P} B\| \left[\frac{\|K_x\|^2_F}{4} + \frac{\|K_r\|^2_F}{4} \right] \end{array} \right\} \tag{15}$$

Eq. (15) essentially means that, once trajectories of $e(t)$ enter the compact set E_0 in finite time, they will remain inside this compact set regardless of what happens. Eq. (13) and Eq. (14) guarantee the boundedness of $\|\Delta K_x\|$ and $\|\Delta K_r\|$, guaranteeing that all signals and adaptive parameters will be bounded.

3 Discussion

This paper will consider the same satellite launcher presented in [5] for simulations. The reference model is calculated so that $\omega_n = 3.5 rad/s$ and $\xi = 0.7$. $\Gamma_x = 100 I_2$ and $\Gamma_r = 100$. $\sigma = 0.1$ in all simulations. When considered, the noise ξ was modeled as uniform random signal in θ, varying between -0.1° and 0.1°.

Fig. 2 and Fig. 3 show a comparison between a traditional reference model adaptive controller (calculated with $Q = 100I_2$) and a closed loop reference model adaptive controller ($Q_0 = 0.1I_2$ and $\nu = 0.7$) without and with noise, respectively. The plant is time variant, and the closed loop reference model seems to provide better results when compared to the traditional one.

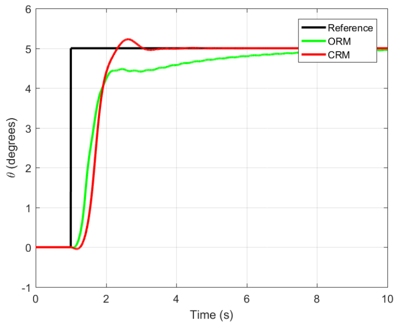

Figure 2: Comparison between traditional reference model and closed loop reference model.

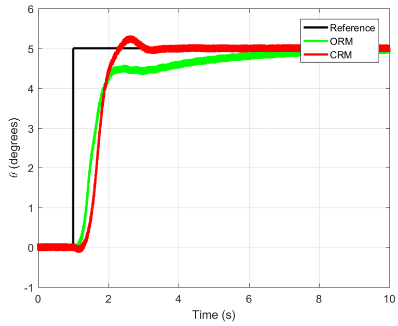

Figure 3: Comparison between traditional reference model and closed loop reference model.

Figs. 4-7 show the system response for different values of ν without noise.

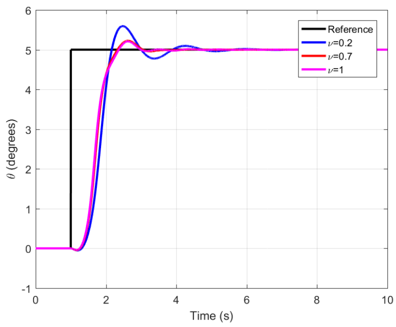

Figure 4: Output.

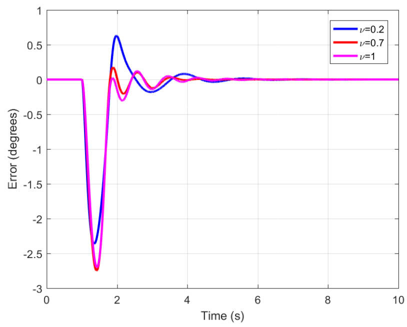

Figure 5: State error.

Figs. 8-11 show the system response for different values of ν with presence of noise. It is possible to see that, when ν is too small, the system becomes more oscillatory.

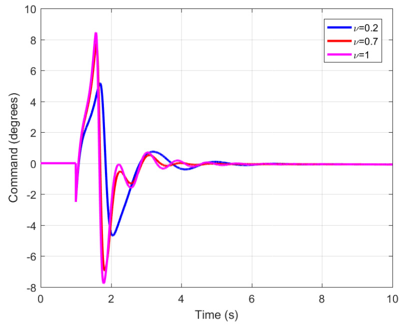

Figure 6: Control signal.

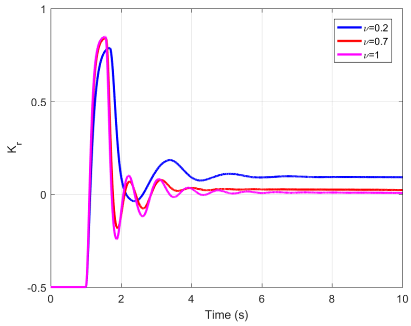

Figure 7: Kr.

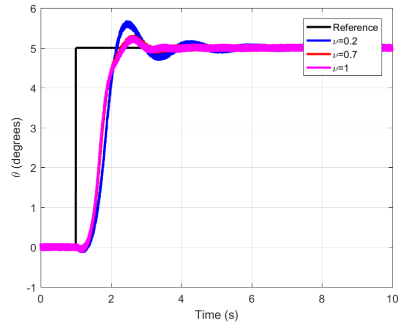

Figure 8: Output.

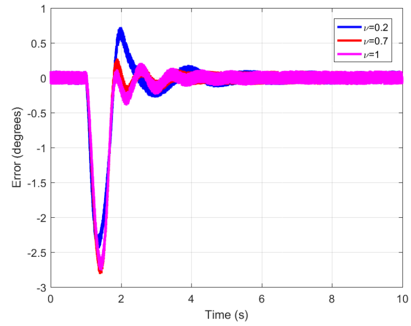

Figure 9: State error.

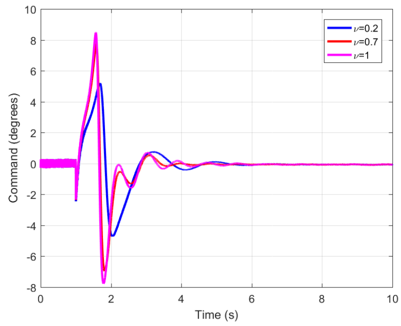

Figure 10: Control signal.

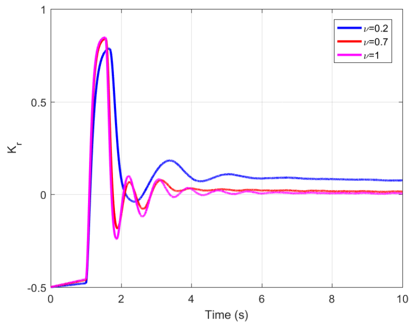

Figure 11: Kr.

4 Conclusion

This paper provided an analysis of the performance of adaptive controllers with a closed loop reference model when applied to satellite launchers. Furthermore, a complete stability analysis was provided in order to guarantee the boundedness of signals and adaptive parameters for a generic case.

When analyzing specific simulation results, it was possible to see that the CRM provided better tracking when dealing with time variant systems, while also providing good responses when dealing with bounded disturbances.

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