A GENERAL SOLUTION OF THE QUASI-STEADY DROPLET COMBUSTION

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Abstract- This work presents an analytical solution for a general quasi-steady model of the gas phase in the droplet vaporization and combustion problems. The resulting form of the conservation equations is a non-linear ordinary differential equations system, whose non-linearities are due to the dependence of the molecular transport coefficients on the temperature plus the chemical reaction term. This system can be solved by proper arrangements leading to the supression of the non-linearities. The general Schvab-Zel'dovich formulation is used to remove the chemical reaction term and the other non-linearity is eliminated by writing the system as a function of a single dependent variable.

Introduction

The relative simplicity in describing the spherical droplet vaporization problem (with combustion or not) is based on the fact that this process is mainly controlled by the gas phase in the neighborhood of the droplet, which has a quasi-steady behavior compared to the liquid phase behavior. This is so if ambient conditions change slowly and are such that the gas density is much smaller than the liquid density because the thermal response characteristic time of the gas phase is of the order of the product of the gas/liquid densities ratio with any characteristic time of the liquid phase (e.g., the heating or the vaporization times, Boghi et all 1985, Williams 1985).

Therefore the quasi-steady character of the gas phase processes explains the success of the steady-state model in describing the droplet vaporization dynamics, as shown by Godsave (1953) and Spalding (1953). The outstanding feature of that model is the establishment of the fact that the square of the droplet radius decreases linearly with time.

Goldsmith and Penner (1954) improved the quasi-steady-state model including the linear dependence of molecular transport coefficients and specific heats on the temperature, leading the processes in the gas phase to be described by a system of non-linear ordinary differential equations. A convenient combination of those equations allowed the non-linearity to be removed so that their final form consisted in a system of linear ordinary differential equations. Kassoy and Williams (1968) generalized that model by including the chemical reaction and the dependence of the transport properties on a power of the temperature. They assumed however constant specific heats and Lewis numbers equal to one. Law (1975) formulated a more general quasi-steady-state model for describing the droplet vaporization with transport properties and specific heats depending on the temperature and the gas composition. The effect of variable transport coefficients on the combustion problem was studied by Raghunandan and Mukunda (1977). They eliminated this non-linearity through a change of variables and avoided the non-linearity caused by the chemical reaction by imposing the knowledge of the flame temperature. Williams (1985) solved the same problem including the chemical reaction. He employed a function defined by a combination of dependent variables to remove the non-linear chemical reaction term (The Schvab-Zel'dovich formulation).

We present in this paper still another way to find the solution of the droplet combustion. We admit a more general quasi-steady model, which includes the dependence of the molecular transport properties on the temperature and the chemical reaction as before, but allowing for non-unity Lewis numbers. To solve this problem we use the general Schvab-Zel'dovich's formulation for eliminating the chemical reaction term and the Goldsmith and Penner's procedure to remove the non-linearity of the transport properties. The liquid phase problem, although formulated, shall not be analized here.

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