DYNAMICS OF FLAME IN NARROW CHANNEL

FINAL REPORT ON SCIENTIFIC INITIATION
(PIBIC/CNPq/INPE)

Giovanna Querido Marcondes de Assis (Universidade Federal de Itajubá, Scholar PIBIC/CNPq)
E-mail: giovannaqmassis@gmail.com

Fernando F. Fachini (LCP/INPE, Professor adviser)
E-mail: fachini@lcp.inpe.br

July, 2014

ABSTRACT

We analyse premixed flames inside a narrow channel.This topic matters the most when it comes to security and propulsion. It's study started in order to analyze flames propagating in coal mines, contributing later on to chemical industries. Intrinsically, the problem is two-dimensional, but applying weight mean procedure, it is possible to describe the problem with 1D conservation equations making usage of convective heat transfer model. Considering the gas and solid phase conjugated problem, the system is adiabatic, then all heat transferred from the flame to the tube walls returns to the fuel mixture. As in most cases the heat conductivity in the solid phase is much faster than that in the gas phase, problem's description requires two main thermal zones with very different length scales. The largest scaled one is associated to solid phase conduction, while the smallest reefers to the gas phase. The analysis focused in showing the influence of these two thermal zones, once their differences contribute to the understanding of the propagation of heat on the channel wall and on the flame. Similar studies on this, however treated in porous media, show that the flame temperature will probably rise above the adiabatic values. Solving the particular equations of mass, energy and species conservation, it is used the method of singular perturbations (asymptotic expansion), mating outcomes for different regions. The perturbation theory provides solutions as series expansions having as parameter the ratio between the thermal conductivities of the solid and gas phases. Other parameters in the mathematical formulation are oxygen and fuel's Lewis numbers and also the convective heat transfer parameter. For conditions in which the convective heat transfer parameter is close to unit, the two phases are not in thermal equilibrium, so the heat circulation between phases happens largely and the flame temperature can reach up to twice the adiabatic temperature (theoretically estimated value). Further, a third region is analyzed along this work. It has a heat-reactive character and its length scale is appropriate to describe the flame inner structure. The inner region study coupled with the outer region one solves the problem.

RESUMO

Neste trabalho estuda-se uma chama pré-misturada estacionária dentro de canais. Este assunto é de interesse quando se trata de seguranca e propulsão. Comecou a ser estudado com o intuito de analisar chamas em minas de carvão, contribuindo, depois, para indústrias químicas. Intrinsecamente, o problema é bidimensional, mas pode ser convertido para uma dimensão considerando o valor médio das variáveis na direo normal às paredes. Considerou-se a condicão adiabática entre as paredes do canal e o meio externo, logo todo calor transferido da chama para as paredes retorna à mistura combustível. Como na maioria dos casos o transporte condutivo de calor na fase sólida é muito mais rápido que aquele na fase gasosa, a descricão do problema demanda a consideracão de duas regiões térmicas de escalas espaciais características muito diferentes, sendo a maior relacionada à conducão no sólido e a menor, conducão no gás. O foco dessa análise é mostrar a influência tanto das duas regiões térmicas, bem como da recirculacão de calor através das paredes do canal no comportamento da chama. Trabalhos similares a este, porém em meios porosos, mostram um aumento na temperatura da chama acima do valor adiabático. Na solucão das equacões de conservacões da massa, energia e espécie na forma adimensional, particularizadas ao problema, emprega-se o método de perturbacões singulares (expansão assimptótica), acoplando-se os resultados para as diferentes regiões. O método de perturbacões gera solucões em forma de séries tendo como parâmetro de expansão a razão entre as condutividades térmicas das fases gasosa e sólida. Os outros parâmetros que aparecem na formulacão matemática são os números de Lewis para o oxigênio e combustível e o parâmetro de transferência de calor convectivo. Nas condicões que impõem um valor ao parâmetro de transferência de calor convectivo próximo de um, as duas fases não estão em equilíbrio térmico, por isso a recirculacão de calor entre as fases é intensa e a temperatura da chama pode chegar a até duas vezes a temperatura adiabática (valor estimado teoricamente). Ainda, uma terceira região é analisada junto com as duas outras. Ela tem um caracter térmico-reativo e sua escala espacial característica é tal que descreve a estrutura interna da chama. Com o estudo desta região e o acoplamento com a região externa, o problema fica resolvido.

SUMMARY

1 INTRODUCTION

Propagation of premixed flame in tube or channel has been studied since the end of XIX century. The main interest is on safety issues. Originally, the safety on coal mines demanded analyses of flame extinction condition to determine the quenching distance. Later, the chemical industries which work with reacting flows asked information to avoid flame propagation inside the tube. Recently with the option of applying detonation for air plain propulsion, studies on flame propagation are also focused on the transition deflagration to detonation. It is worth to note that, the detonation condition depends on the initiation of the flame propagation on tubes or channels.

In the current problem flame propagation happens in a channel. The flow inside the channel can be devided, in a first view, in two according its thermal regime. Upstream the flame, the flow is at low temperature and close to the flame, the temperature is higher than that of the wall temperature. However, far from the flame the wall temperature is higher than that of the flow. Downstream the flame, the flow and the channel wall is at thermal equilibrium. The fluid mechanics problem in the gas phase and thermal problem in both phases are described by the mass, momentum, species and energy conservation equations. In this view, the flame is a infinitely thin surface with infinite chemical reaction rate, then the flame temperature is described by the velocity of the flow.

In general, the thermal conductivity of the solid phase is much larger than that of the gas phase. By this reason, the upstream of the flame can be divided in two regions. The larger one is that controlled by the thermal problem in the solid phase and the smaller one is that controlled by the thermal problem in the gas phase. The larger region is denoted by outer zone and the smaller region, adjacent to the flame, is denoted by inner zone.

2 LENGTH-SCALES AND DESCRIPTION OF THE PROBLEM

Heat is transfered from the flame to a thin layer of unburnt gas upstream it, to the channel walls and then back to incoming fresh gas.This heat recirculation allows the flame to reach higher temperatures than those in freely propagating flames.

It is remarkable the difference between gas and solid phases properties. Solid thermal conductivity can be several times larger than that for gas-phase; for example, we have the combination steel ($K_s = 52W/mK$)-air($K_g = 0.023W/mK$). This condition results in discrepant differences between solid and gas temperatures near the flame. Also, because of this property, Γ can be defined in advance as he ratio of the solid and the gas-phase conductivities ($\Gamma \equiv K_s/K_g$).

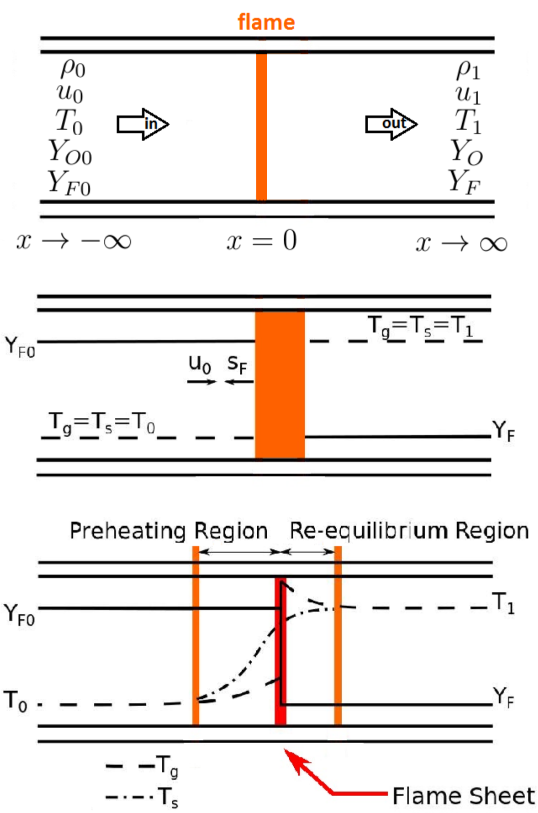

Figure 1: Problem schema and temperatures and fuel mass fraction distributions for different length-scales.

In a first sight, channel walls heat the unburnt gas in a large region upstream the flame. It is called first preheating region. In this large reagion, the two dominant processes are the heat transfer by the solid walls and the heat exchange between the solid and gas phase. Coming close to the flame into the small region, the heat transfer by the gas phase becomes of the same order of magnitude comparing to the two mentioned processes. Hence, it is importante to determine the local temperatures. As already mentioned, downstream the flame, the solid and gas phase are in thermal equilibrium.

The characteristic length scale(solid) of the region controled by the heat transfer by the solid walls and the heat exchange between the solid and gas phase is given by the ratio between solid-phase conductive heat transfer and gas-phase heat convection, $l_s = K_s/\rho_0 u_0 c_p$. In addition, the characteristic length scale (gas) of the region controled by the conductive heat transfer by the gas phase and gas-phase heat convection, $l_g = K_g/\rho_0 u_0 c_p$.

It is important to mention that the difference between l_s and l_g is the ratio between the solid and gas thermal conductivity, $\Gamma = K_s/K_g$.

The present problem consists in a stationary flame, for this, $s_F = u_0$, the flame and incoming gas velocities should be equal in magnitude, but different signs. Upstream from the the flame, solid and gas temperatures, such as fuel mass fraction are equal to their incoming values (T_0 e Y_{F0}). Unlikely, closer to the flame, both, unburnt gas and solid walls, reach adiabatic flame temperature T_1, while fuel mass fraction, naturally, reaches zero, for lean mixtures. At the flame sheet, gas temperature gets to the superadibatic peak, T_{sup}, but quickly returns to its adiabatic temperature T_1.

In this work,the transport and thermodynamic properties are constant, such as pressure inside the channel. We are considering a reacting lean mixture, formed by oxidant initial mass fraction Y_{O0} and fuel initial mass fraction Y_{F0}. The flow characteristics are ρ_0 as the unburnt gas-phase density and u_0 as the income velocity.

From global energy conservation, combustion products have temperature $T_1 = T_0 + Y_{F0}Q/C_p$, where Q is the heat released in the flame and C_p is the gas-phase heat capacity.

3 MATHEMATICAL MODEL

First of all, are presented the conservation equations and then they will be non-dimensionalized. Due to the large discrepance on the characteristic length scales to the both thermal problems uptream the flame, it is possible to employ the asymptotic expansion method to find the problem solution.

According to characteristic of the flow inside channel or tube, the problem is described by a two dimensional conservation equations system.

Combustion is assumed to occur following a global one-step mechanism, represented in mass unit as

$$F + sO_2 \rightarrow (1+s)P,$$

in which s is the ratio between mass of oxygen and mass of fuel in a stoichiometric reaction.

Therefore, the steady-state conservation equations are:

Mass Conservation:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{1}$$

Conservation of Momentum:

$$\frac{\partial}{\partial x_i}[\rho u_i(u,v)] = -\left(\frac{\partial}{\partial x}p, \frac{\partial}{\partial y}p\right) + \mu\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right](u,v) + \rho(g_x, g_y) \tag{2}$$

Energy Conservation:

$$\frac{\partial}{\partial x_i}[\rho(u,v)e] =$$

$$\rho(u,v)(g_x,g_y) - \left(\frac{\partial}{\partial x}pu, \frac{\partial}{\partial y}pv\right) + \tau\frac{\partial}{\partial x_i}(u,v) + K\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] + Qw \tag{3}$$

As we are looking to a horizontal tube,gravity variations do not apply. Also, the tube is opened on both sides, so the pressure term should be omitted. With these considerations, we have(in 2 D):

$$\frac{\partial}{\partial x}(\rho u C_p T) + \frac{\partial}{\partial y}(\rho v C_p T) = K\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] + Qw \tag{4}$$

Species Conservation:

$$\frac{\partial}{\partial x}(\rho u Y_i) + \frac{\partial}{\partial y}(\rho v Y_i) = \nabla(\rho D \nabla Y_i) - s_i w \tag{5}$$

Now, applying the weight average on these laws,integrating them in y, they will return to their 1D condition:

Mass:

$$\int \frac{\partial}{\partial x}(\rho \vec{u}) + \frac{\partial}{\partial y}(\rho \vec{v})\, dy = 0$$

so, we have:

$$\int_0^L \rho u dy = C = \dot{m} = \rho_0 u_0 \tag{6}$$

Energy:

$$\int \frac{\partial}{\partial x}(\rho u C_p T) + \frac{\partial}{\partial y}(\rho v C_p T) dy = \int_0^L \frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) dy + \int Qw dy$$

results in:

$$C_p \dot{m}\frac{d\overline{T}}{dx} = \frac{d}{dx}\left(K\frac{d\overline{T}}{dx}\right) + h(T_p - T) + Q\dot{W} \tag{7}$$

where

$$T = T_g$$

$$\int_0^L w dy = \dot{W} \equiv A\rho^2 Y_o Y_F e^{-E/RT}$$

$$K\frac{dT}{dy}|_0^L = h(T_p - T_g) = h(T_p - T)$$

If

$$\int \rho u T dy = \int \rho u dy \int T dy$$

is verified and choosing $\overline{T} = \int_0^L T dy$.

Energy (Channel wall - solid):

$$\int_L^{L+h} \frac{\partial}{\partial x}(\rho u C_p T) + \frac{\partial}{\partial y}(\rho v C_p T) dy =$$

$$\int_L^{L+h} \frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) dy + \int_L^{L+h} Q w dy$$

as the solid do not move, it doesn't have the speed term (u,v), also chemical reactions are not shown in the solid equations. That gives us:

$$\frac{\partial}{\partial x}\left(K_g \frac{\partial}{\partial x}\bar{T}_p\right) - h(\bar{T}_p - \bar{T}) = 0 \tag{8}$$

Species:

$$\int_0^L \frac{\partial}{\partial x}(\rho u Y_i) + \frac{\partial}{\partial y}(\rho v Y_i) dy = \int_0^L \nabla(\rho D \nabla Y_i) - s_i w dy$$

in which i index is the indication for Oxidant or Fuel.

Since there is nothing flowing on vertical direction and the model is symmetric, velocity and mass fraction derivatives are null.

$$\dot{m}\frac{d}{dx}\bar{Y}_i = \frac{d}{dx}\left(D\frac{d\bar{Y}_i}{dx}\right) - s_i \dot{W} \tag{9}$$

where, $\bar{Y}_i = \int_0^L Y_i dy$.

3.1 Non-dimensionalization

Definining the non-dimensional variables

$$y_F \equiv \frac{\bar{Y}_F}{Y_{F0}}, \quad y_O \equiv \frac{\bar{Y}_O}{Y_{O0}}, \quad \theta \equiv \frac{Cp(\overline{T} - T_0)}{Y_{F0} Q} = \frac{\overline{T} - T_0}{T_1 - T_0}$$

Then, Eq. (7) becomes:

$$\frac{d\theta_g}{d\mathrm{x}} = \frac{1}{\Gamma}\frac{d^2\theta_g}{d\mathrm{x}^2} + \Gamma \mathrm{Da} \mathrm{y}_0 \mathrm{y}_\mathrm{F} e^{\left[-\frac{\beta(1-\theta_g)}{1-\hat{\alpha}(1-\theta_g)}\right]} + N(\theta_s - \theta_g) \tag{10}$$

Equation (8) becomes:

$$\frac{d^2\theta_s}{d\mathrm{x}^2} - N(\theta_s - \theta_g) = 0 \tag{11}$$

Equation (9) for $i = F$, F stands for fuel, becomes

$$\frac{d}{d\mathrm{x}}\mathrm{y}_\mathrm{F} = \frac{1}{Le_F\Gamma}\frac{d^2}{d\mathrm{x}^2}\mathrm{y}_\mathrm{F} - \Gamma \mathrm{Da} \mathrm{y}_\mathrm{O} \mathrm{y}_\mathrm{F} e^{\left[-\frac{\beta(1-\theta_g)}{1-\hat{\alpha}(1-\theta_g)}\right]} \tag{12}$$

in which $Le_F = K_g/\rho C_p D_F$.

Rearranging Eq. (9) for $i = O$, O stands for oxidant,

$$\frac{d}{d\mathrm{x}}\mathrm{y}_\mathrm{O} = \frac{1}{Le_O\Gamma}\frac{d^2}{d\mathrm{x}^2}\mathrm{y}_\mathrm{O} - S\Gamma \mathrm{Da} \mathrm{y}_\mathrm{O} \mathrm{y}_\mathrm{F} e^{\left[-\frac{\beta(1-\theta_g)}{1-\hat{\alpha}(1-\theta_g)}\right]} \tag{13}$$

in which $S \equiv Y_{F0}s/Y_{O0}$ and $Le_O = K_g/\rho C_p D_O$.
The parameters used were defined by:

$$\Gamma \equiv \frac{K_s}{K_g}, \quad \hat{\alpha} \equiv \frac{(T_1 - T_0)}{T_1}, \quad \beta \equiv \frac{E(T_1 - T_0)}{R{T_1}^2},$$

$$\mathrm{Da} \equiv \frac{A\rho^2 Y_{O0} e^(- \beta/\hat{\alpha})}{\rho_0^2 u_0^2 Cp}, \quad N \equiv \frac{K_s h}{(\rho_0 u_0 Cp)^2}.$$

Dimensionless parameter $\hat{\alpha}$ measures heat release, β is the Zel'dovich number, Le_F and Le_O are Lewis numbers, Da is the Damkhöler number and N measures heat transfered between phases.

In this problem $N \gg 1$, proposing that interphase heat transfer happens widely.

3.2 Outer zone resolution: solution of the order of unity

In the outer zone, reaction is not considered, once it is defined before the flame sheet. Hence, Eqs. (10) to (13) take the form:

$$\frac{d\theta_g}{d\mathrm{x}} = \frac{1}{\Gamma}\frac{d^2\theta_g}{d\mathrm{x}^2} + N(\theta_s - \theta_g) \tag{14}$$

$$0 = \frac{d^2\theta_s}{d\mathrm{x}^2} - N(\theta_s - \theta_g) \tag{15}$$

$$\frac{d}{d\mathrm{x}}\mathrm{y_i} = \frac{1}{Le_i\Gamma}\frac{d^2}{d\mathrm{x}^2}\mathrm{y_i} \tag{16}$$

where i stands for fuel F and oxygen O.

Then, we can start solving the problem. The first condition to the current problem is $N \gg 1$, as the other terms in the equation are of the order of unity, in first approximation those unitary terms are neglected which leads to $\theta_g = \theta_s$. Then, combining Eqs. (14) and (15) and using $\theta = \theta_g = \theta_s$,

$$\theta = \frac{1}{\Gamma}\frac{d\theta}{d\mathrm{x}} + \frac{d\theta}{d\mathrm{x}} + C \tag{17}$$

Using the boundary condition $\theta = 0$, for $x \to -\infty$, leads to $C = 0$ in the region $x < x_f$. Using the boundary condition $\theta = 1$, for $x \to \infty$, leads to $C = 1$ in the region $x > x_f$.

It is searched solution for θ in the form of series,

$$\theta = \theta_0 + \Gamma^{-1}\theta_1 + \Gamma^{-2}\theta_3 + o(\Gamma^{-2})$$

Taking θ into Eq. (17) and separating the terms of power of Γ^{-1}, the leading order problem is

$$\frac{d\theta_0}{d\mathrm{x}} - \theta_0 = -C \tag{18}$$

Since the solution of the homogeneous equation is $e^{(x-x_f)}$ and the temperature in the region down stream the flame is limited, the solution of (18) for $x > x_f$ is $\theta = 1$, then,

$$\theta_0 = \begin{cases} e^{(x-x_f)}, & \text{for } x < x_f \\ 1, & \text{for } x > x_f \end{cases} \tag{19}$$

The first order problem is

$$\frac{d\theta_1}{d\mathrm{x}} - \theta_1 = -\frac{d\theta_0}{d\mathrm{x}} \tag{20}$$

and the solutions are

$$\theta_1 = \begin{cases} -(x-x_f)e^{(x-x_f)}, & \text{for } x < x_f \\ 0, & \text{for } x > x_f \end{cases} \tag{21}$$

Furthermore, for species **conservation equations** (16), the leading order term gives us:

$$\frac{d}{dx} y_{i0} = 0$$

Then, the mass fraction upstream the flame is $y_{i0} = C_1 = 1$ because of $y_{i0} = 1$ as $x \to -\infty$. Consequentely, all solutions for upper order terms are zero, $y_{i1} = y_{i2} = \cdots = 0$.

3.3 First inner zone resolution: solution of the order of unity

To describe the thermal problem in the gas phase characteristic length scale, controled by the conductive heat transfer by the gas phase and gas-phase heat convection, it is necessary to perform the following spatial transformation $\hat{x} = (x - x_f)\Gamma$. The same symbol $\hat{(\ )}$ is used in the other variables to identify them as those describing this region.

$$\Gamma \frac{d\hat{\theta}_g}{d\hat{x}} = \Gamma \frac{d^2\hat{\theta}_g}{d\hat{x}^2} + \Gamma N_0(\hat{\theta}_s - \hat{\theta}_g) \tag{22}$$

$$0 = \Gamma^2 \frac{d^2\hat{\theta}_s}{d\hat{x}^2} - \Gamma N_0(\hat{\theta}_s - \hat{\theta}_g) \tag{23}$$

$$\Gamma \frac{dy_i}{d\hat{x}} = \frac{\Gamma}{Le_i} \frac{d^2 y_i}{d\hat{x}} \tag{24}$$

Inside the first inner zone, boundary conditions are determined when the outer zone solution is matched with the inner zone problem of the order of unity. Performing the change of variables $\hat{x} = (\mathrm{x} - \mathrm{x_f})\Gamma$, the boundary conditions are:

$$\begin{aligned} &\frac{d\hat{\theta}_{g0}}{d\hat{x}} = \frac{d\hat{\theta}_{s0}}{d\hat{x}} = 0, \\ &\frac{d\hat{\theta}_{g1}}{d\hat{x}} = \frac{d\hat{\theta}_{s1}}{d\hat{x}} = 1, \\ &\hat{\mathrm{y_F}} = \hat{\mathrm{y_O}} = 1, \text{ for } \hat{x} \to -\infty \end{aligned} \tag{25}$$

$$\begin{aligned} &\frac{d\hat{\theta}_{g1}}{d\hat{x}} = \frac{d\hat{\theta}_{s1}}{d\hat{x}} = 0, \\ &\hat{\mathrm{y_F}} = 0 \text{ and } \hat{\mathrm{y_O}} = 1 - \phi, \text{ for } \hat{x} \to \infty, \end{aligned} \tag{26}$$

where ϕ is the equivalence ratio ($\equiv sY_F/y_O$).

For this problem, we will try to find a solution as series:

$$\begin{aligned} \hat{\theta}_g &= \hat{\theta}_{g0} + \Gamma^{-1}\hat{\theta}_{g1} + \Gamma^{-2}\hat{\theta}_{g2} + \dots \\ \mathrm{y_i} &= \mathrm{y_{i0}} + \Gamma^{-1}\mathrm{y_{i1}} + \Gamma^{-2}\mathrm{y_{i2}} + \dots \\ \hat{\theta}_S &= 1 - \Gamma^{-1}\hat{\theta}_{s1} + \Gamma^{-2}\hat{\theta}_{s2} + \dots \end{aligned} \tag{27}$$

Substitute the solution Eq. (27) into Eq. (22) and defining $\epsilon = 1/\Gamma$, we get:

$$\frac{d}{d\hat{\mathrm{x}}}(\hat{\theta}_{g0}+\epsilon\hat{\theta}_{g1}+\epsilon^2\hat{\theta}_{g2}) = \frac{d^2}{d\hat{x}^2}(\hat{\theta}_{g0}+\epsilon\hat{\theta}_{g1}+\epsilon^2\hat{\theta}_{g2})+N_0(1-\epsilon\hat{\theta}_{s1}+\epsilon^2\hat{\theta}_{s2}-(\hat{\theta}_{g0}+\epsilon\hat{\theta}_{g1}+\epsilon^2\hat{\theta}_{g2})) \tag{28}$$

Making Eq.(28) $\equiv F(\hat{\theta};\epsilon)$ and taking the limit $\epsilon \to 0$, we shall have for the leading order:

$\lim_{\epsilon\to 0} F(\hat{\theta}) \Rightarrow \hat{\theta}''_{g0} - \hat{\theta}'_{g0} - N_0(\hat{\theta}_{g0} - 1) = 0$

For which the solution is:

$\hat{\theta}_{g0} = C_1 e^{\frac{1+\sqrt{1+4N_0}\hat{x}}{2}} + C_2 e^{\frac{1-\sqrt{1+4N_0}\hat{x}}{2}} + 1$

In order to find C_1 and C_2, the problem should be separated in two situations $\hat{x} < 0$ and $\hat{x} > 0$. By using the boundary conditions, $\hat{\theta}_{g0}$ as $\hat{x} \to -\infty$ should match the profile of θ_{g0} for $x = x_f$. Then, we have for the order of unity solution:

$$\begin{cases} \hat{\theta}_{g0} = 1 - (1 - \hat{\theta}_{g0f})e^{r_1\hat{x}}, & \text{for} \quad \hat{x} < 0 \\ \hat{\theta}_{g0} = 1 - (1 - \hat{\theta}_{g0f})e^{r_2\hat{x}}, & \text{for} \quad \hat{x} > 0 \end{cases} \tag{29}$$

where $\hat{\theta}_{g0f}$ is the gas temperature inside the flame.

Applying the same changes of variables as previously, Eq. (23) takes the form

$$0 = \Gamma^2 \frac{d^2\hat{\theta}_s}{d\hat{x}^2} - N_0\Gamma(\hat{\theta}_s - \hat{\theta}_g) \tag{30}$$

$\hat{\theta}_s$ and $\hat{\theta}_g$ were replaced for the series solution, as happened in Eq.(28). At this point, the equation for $\hat{\theta}_s$ should match the solution for θ_s, for $\hat{x} \to -\infty$. By doing this, it is assured that the heat flux will be continuos in the solid phase, so $d\hat{\theta}_s/d\hat{x}$ should be equal to $\Gamma^{-1}d\theta_s/dx$ at the flame .This process resulted in a equation which we called $S(\hat{\theta})$. Over again, limit will be used to find the order of unity solution.

$\lim_{\epsilon\to 0} S(\hat{\theta}) \Rightarrow 0 = \hat{\theta}''_{s1} - N_0(1 - \hat{\theta}_{g0})$

$$
\begin{cases}
\hat{\theta}''_{s1} - N_0(1-\hat{\theta}_{g0f})e^{r_1\hat{x}} = 0, & \text{for} \quad \hat{x} < 0 \\
\\
\hat{\theta}''_{s1} - N_0(1-\hat{\theta}_{g0f})e^{r_2\hat{x}} = 0, & \text{for} \quad \hat{x} > 0
\end{cases}
\tag{31}
$$

where

$$
r_1 \equiv \frac{1+\sqrt{1+4N_0}}{2}
$$

$$
r_2 \equiv \frac{1-\sqrt{1+4N_0}}{2}
$$

Thus, using boundary conditions described in Eqs. (25) e (26), integrating Eq. (31) and pushing $\hat{x}$ to extremes $-\infty$ and ∞, we get:

$$
\begin{cases}
\hat{\theta}'_{s1} = 1 + \dfrac{N_0(1-\hat{\theta}_{g0f})e^{r_1\hat{x}}}{r_1}, & \text{for} \quad \hat{x} < 0 \\
\hat{\theta}'_{s1} = \dfrac{N_0(1-\hat{\theta}_{g0f})e^{r_2\hat{x}}}{r_2}, & \text{for} \quad \hat{x} > 0
\end{cases}
\tag{32}
$$

Integrating again, we find the solution for $\hat{\theta}_{s1}$.

$$
\begin{cases}
\hat{\theta}_{s1} = \hat{x} + \dfrac{N_0(1-\hat{\theta}_{g0f})e^{r_1\hat{x}}}{r_1^2} + C_3, & \text{for} \quad \hat{x} < 0 \\
\hat{\theta}_{s1} = \dfrac{N_0(1-\hat{\theta}_{g0f})e^{r_2\hat{x}}}{r_2^2}, & \text{for} \quad \hat{x} > 0
\end{cases}
\tag{33}
$$

Finally, the values found for the position $\hat{x} = 0$ must converge for solutions coming up and downstream the flame.
If we check $\hat{\theta}_{s1}$ derivatives for the inner zone, we get:

$$
\begin{cases}
\hat{\theta}'_{s1} = 1 + \dfrac{N_0(1-\hat{\theta}_{g0f})}{r_1}, & \text{for} \quad \hat{x} \to 0^- \\
\hat{\theta}'_{s1} = \dfrac{N_0(1-\hat{\theta}_{g0f})}{r_2}, & \text{for} \quad \hat{x} \to 0^+
\end{cases}
$$

$$
\therefore 1 + \frac{N_0(1-\hat{\theta}_{g0f})}{r_1} = \frac{N_0(1-\hat{\theta}_{g0f})}{r_2}
$$

Which, solving, gives us:

$$
(\hat{\theta}_{g0f}) = 1 - \frac{r_1 r_2}{N_0(r_1 - r_2)} = 1 + \frac{1}{\sqrt{1+4N_0}}
\tag{34}
$$

It proves that the temperature at the flame is higher than 1 (adiabatic temperature).

In fact, if N_0 approximates to 0, temperature $\hat{\theta}_{g0f}$ could reach 2 times the adiabatic temperature.
Checking $\hat{\theta}_{s1}$:

$$\hat{\theta}_{s1} = \frac{N_0(1-\hat{\theta}_{g0f})}{r_1^2} + C_3, \qquad \text{for} \quad \hat{x} \to 0^-$$

$$\hat{\theta}_{s1} = \frac{N_0(1-\hat{\theta}_{g0f})}{r_2^2}, \qquad \text{for} \quad \hat{x} \to 0^+$$

$$\therefore \frac{N_0(1-\hat{\theta}_{g0f})}{r_1^2} + C_3 = \frac{N_0(1-\hat{\theta}_{g0f})}{r_2^2}$$

$$C_3 = \frac{\sqrt{1+4N_0}(1-\hat{\theta}_{g0f})}{N_0} = -\frac{1}{N_0}$$

For species conservation, substituting series solution, Eq. (24), into Eq. (23), defining $\epsilon = 1/\Gamma$ and making $\epsilon \to 0$, as it was done for Eq. (27), one may find:

$$\hat{\mathrm{y_{i0}}}'' - Le_i \hat{\mathrm{y_{i0}}}' = 0.$$

For which the solution is:

$$\hat{\mathrm{y_F}} = C_4 e^{Le_F(\hat{x}-\hat{x_f})} + C_5$$

$$\hat{\mathrm{y_O}} = C_6 e^{Le_O(\hat{x}-\hat{x_f})} + C_5$$

Using boundary conditions described in Eqs. (25) and (26), we shall have

$$\begin{cases} \hat{\mathrm{y_{i0}}} = C_5 = 1, & \text{for} \quad \hat{x} \to -\infty \\ \\ \hat{\mathrm{y_F}} = C_4 = -1, & \\ \hat{\mathrm{y_O}} = C_6 = -\phi, & \text{for} \quad \hat{x} \to \hat{x_f} \end{cases}$$

The solutions here described are for $N = N_0\Gamma$. For this order, flame velocities are smaller and heat transfer between the phases become relevant.

4 CONCLUSIONS

Although this project has not been finished yet, some results were already presented. The order of unity solution was given considering a solid-phase diffusion length-scale (l_s), in which the solid-phase heat conduction and gas-phase convection dominated the problem. Also, the interphase heat transfer was considered highly substantial, generating high temperatures, possibly, for low flame speed. Inside the inner zone, where gas-phase heat convection and conduction are of the same order, we used the gas-lenght scale l_g. By using the results found for the outer zone, when they approach zero (equivalent to inner zone region), and going on the opposite way (from inner to outer zone), we were able to find an expression for the gas temperature inside the flame. Finally, it was concluded that this temperature peak is higher than the adiabatic temperature. If the interphase heat parameter (N_0) approaches zero, inside the flame, the gas temperature can reach up to two unities.

REFERENCES

[1] PEREIRA, F.M.; OLIVEIRA, A.A.M.; FACHINI, F. F. Asymptotic analysis of stationary adiabatic premixed flames in porous inert media, **Combustion and Flame**, 156 (2009) 2283-2288.

[2] PEREIRA, F.M.; OLIVEIRA, A.A.M.; FACHINI, F. F. Theoretical analysis of ultra-lean premixed flames in porous inert media, **J. Fluid Mech.**, 657(2010) 285-307.