Monthly Notices
of the
ROYAL ASTRONOMICAL SOCIETY

MNRAS **492,** 5949–5955 (2020)
Advance Access publication 2020 January 25

doi:10.1093/mnras/staa205

Gravitational waves from fast-spinning white dwarfs

Manoel F. Sousa,[1⋆] Jaziel G. Coelho[1,2⋆] and José C. N. de Araujo[1⋆]

[1] *Divisão de Astrofísica, Instituto Nacional de Pesquisas Espaciais, Avenida dos Astronautas 1758, São José dos Campos, SP 12227-010, Brazil*
[2] *Departamento de Física, Universidade Tecnológica Federal do Paraná, 85884-000 Medianeira, PR, Brazil*

Accepted 2020 January 22. Received 2019 December 26; in original form 2019 November 5

ABSTRACT

Two mechanisms of gravitational waves (GWs) emission in fast-spinning white dwarfs (WDs) are investigated: accretion of matter and magnetic deformation. In both cases, the GW emission is generated by an asymmetry around the rotation axis of the star. However, in the first case, the asymmetry is due to the amount of accreted matter on the magnetic poles, while in the second case it is due to the intense magnetic field. We have estimated the GW amplitude and luminosity for three binary systems that have a fast-spinning magnetized WD, namely, AE Aquarii, AR Scorpii, and RX J0648.0−4418. We find that, for the first mechanism, the systems AE Aquarii and RX J0648.0−4418 can be observed by the space detectors BBO and DECIGO if they have an amount of accreted mass of $\delta m \geq 10^{-5}$ M$_{\odot}$. For the second mechanism, the three systems studied require that the WD have a magnetic field above $\sim 10^9$ G to emit GWs that can be detected by BBO. We also verified that, in both mechanisms, the gravitational luminosity has an irrelevant contribution to the spin-down luminosity of these three systems. Therefore, other mechanisms of energy emission are needed to explain the spin-down of these objects.

Key words: gravitational waves – stars: magnetic field – white dwarfs.

1 INTRODUCTION

There is an increasing interest of the astrophysics community on highly magnetized white dwarfs (HMWDs) both from the theoretical and observational points of view. These white dwarfs (WDs) with surface magnetic fields from 10^6 up to 10^9 G have been confirmed by the recent results of the Sloan Digital Sky Survey (SDSS; see e.g. Külebi et al. 2009; Kepler et al. 2010, 2013, 2015). Besides their high magnetic fields, most of them have been shown to be massive, and responsible for the high-mass peak at 1 M$_{\odot}$ of the WD mass distribution; for instance: REJ 0317−853 has $M \approx 1.35$ M$_{\odot}$ and $B \approx (1.7\text{–}6.6) \times 10^8$ G (Barstow et al. 1995; Külebi et al. 2010); PG 1658+441 has $M \approx 1.31$ M$_{\odot}$ and $B \approx 2.3 \times 10^6$ G (Liebert et al. 1983; Schmidt et al. 1992); and PG 1031+234 has the highest magnetic field $B \approx 10^9$ G (Schmidt et al. 1986; Külebi et al. 2009). The existence of ultramassive WDs has been revealed in several studies (see e.g. Althaus et al. 2005, 2007; Castanheira et al. 2013; Hermes et al. 2013; Curd et al. 2017; Gentile Fusillo et al. 2018; Jiménez-Esteban et al. 2018; Camisassa et al. 2019).

Typically, WDs rotate with periods of days or even years. Recently, a pulsating WD was discovered, called AR Scorpii, that emits from X-ray to radio wavelengths, pulsing in brightness with a period of 1.97 min (Marsh et al. 2016). The spin-down power is an order of magnitude larger than the observed luminosity (dominated by the X-rays), which together with the absence of obvious signs of accretion suggests that AR Sco is primarily rotation-powered. Furthermore, other sources have been proposed as candidates of WD pulsars. A specific example is AE Aquarii, the first WD pulsar identified, with a short rotation period of $P = 33.08$ s (Terada et al. 2008). On the other hand, the *X-ray Multimirror Mission* (*XMM*)–*Newton* satellite has observed a WD faster than AE Aquarii. Mereghetti et al. (2009) showed that the X-ray pulsator RX J0648.0−4418 (RX J0648, hereafter) is a massive WD with mass $M = 1.28$ M$_{\odot}$ and radius $R = 3000$ km (see Althaus et al. 2005, 2007, for derived mass–radius relations for massive oxygen–neon WDs that predict this radius), with a very fast spin period of $P = 13.2$ s, that belongs to the binary system HD 49798/RX J0648.0−4418.

On the other hand, direct observations of gravitational waves (GWs) have recently been made by LIGO and Virgo. As is well known, the first event was detected in 2015 by LIGO (Abbott et al. 2016). This event, named GW150914, came from the merging of two black holes of masses ∼35.6 and 30.6 M$_{\odot}$ that resulted in a black hole of mass ∼63.1 M$_{\odot}$. Thereafter, LIGO in collaboration with Virgo observed nine more such events (Abbott et al. 2017a,b,d, 2019). In addition, the event GW170817 reports the first detection of GWs from a binary neutron star inspiral (Abbott et al. 2017c).

All GW detections are within a frequency band ranging from 10 to 1000 Hz, which is the operating band of LIGO and Virgo. As is well known, there are proposed missions for lower frequencies, such as LISA (Amaro-Seoane et al. 2017; Robson, Cornish & Liug

---

⋆ E-mail: manoelfelipesousa@gmail.com (MFS); jazielcoelho@utfpr.edu.br (JGC); jcarlos.dearaujo@inpe.br (JCNdA)

Published by Oxford University Press on behalf of the Royal Astronomical Society

2019), whose frequency band is of $(10^{-4} - 0.01)$ Hz, BBO (Harry et al. 2006; Yagi & Seto 2011), and DECIGO (Kawamura et al. 2006; Yagi & Seto 2017) in the frequency band ranging from 0.01 to 10 Hz.

Different possibilities of generation of continuous GWs have already been proposed (see e.g. Bonazzola & Gourgoulhon 1996; De Araujo, Coelho & Costa 2016a,b, 2017; Franzon & Schramm 2017; Gao, Cao & Zhang 2017; Mukhopadhyay, Rao & Bhatia 2017; Pereira, Coelho & de Lima 2018; De Araujo et al. 2019, and references therein). More recently, Kalita & Mukhopadhyay (2019) show that continuous GWs can be emitted from rotating magnetized WDs and will possibly be detected by the upcoming GW detectors such as LISA, DECIGO, and BBO. Here, we explore two mechanisms of gravitational radiation emission in fast-spinning magnetized WD: accretion of matter and magnetic deformation. In both cases, the GW emission is generated by asymmetry around the rotation axis of the star.

This paper is organized as follows: in Section 2, we describe the two mechanisms of GW emission by deducting the equations for gravitational amplitude and luminosity. In Section 3, we present the calculations applied to three binary systems that have a fast-spinning magnetized WD: AE Aqr, AR Sco, and RX J0648. Moreover, we discuss the results obtained in this section. Finally, in Section 4 we summarize the main conclusions and remarks.

2 GRAVITATIONAL EMISSION MECHANISMS

WDs might generate GWs whether they are not perfectly symmetric around their rotation axes. This asymmetry can occur due to the accretion of matter (Choi & Yi 2000) or due to the huge dipole magnetic field that can make the star become oblate (Chandrasekhar & Fermi 1953). In this work, we analyse the emission of gravitational radiation from fast magnetized WDs by these two mechanisms.

2.1 Accretion of matter

Here, we show the luminosity and the amplitude of the GW for the case of a WD accreting matter via the magnetic poles, which do not coincide with the rotation axis of the star. In this scenario, the secondary star of the system transfers matter to the WD via an accretion column, without forming a disc, and accumulating an amount of mass on the magnetic poles (see Lubow & Shu 1975; Welsh, Horne & Gomer 1998; Hellier 2001; Warner 2003, and references therein).

This configuration has been considered by Choi & Yi (2000) to obtain AE Aqr's gravitational counterpart, where they assume that the spin-down energy is not directly connected to any observable electromagnetic emission, but the high spin-down would be caused by the accreted matter that slowly spreads over the star's surface and generates gravitational radiation. It is worth mentioning that we do not consider that all spin-down is due to GWs and we have established different values for the amount of mass to calculate the observable GW amplitude.

Thus, we consider a rigid object, whose axes of symmetry are (x_1, x_2, x_3), and the corresponding main moments of inertia are I_1, I_2, and I_3, respectively. This solid rotates with angular velocity ω with respect to an axis that makes an angle θ with the x_3-axis. Moreover, we consider that the magnetic dipole axis is also given by the x_3-axis.

With this configuration and doing $I_1 = I_2$, the gravitational amplitude and luminosity are given, respectively, by (see e.g. Shapiro & Teukolsky 1983; Maggiore 2008)

$$h_{\rm ac} = \frac{4G}{c^4}\frac{(I_1 - I_3)\omega^2}{r}\sin^2\theta, \tag{1}$$

and

$$L_{{\rm GW}_{\rm ac}} = -\frac{2}{5}\frac{G}{c^5}(I_1 - I_3)^2\omega^6\sin^2\theta(16\sin^2\theta + \cos^2\theta), \tag{2}$$

where r is the distance to the emitting source.

Now, to determine the moments of inertia I_1 and I_3, we consider that the object has deformities or an amount of mass accumulated about the x_3-axis. We reduce this system to a large sphere with two smaller spheres of matter on the x_3-axis: one at each of the poles of the larger sphere. This would be equivalent to a WD accreting matter by the two magnetic poles, where the magnetic poles do not coincide with the rotation axis of the star. Therefore, it follows immediately that

$$I_1 = \frac{2}{5}MR^2 + 2\delta m\ R^2, \tag{3}$$

$$I_3 = \frac{2}{5}MR^2 + 2\frac{2}{5}\delta m\ a^2, \tag{4}$$

where M is the mass of the star, R is the radius of the star, δm is the amount of mass accumulated on one magnetic pole, and a is the radius of this amount.

Considering that $R \gg a$, the term $I_1 - I_3$ can be expressed as follows:

$$I_1 - I_3 = 2\delta m\ R^2. \tag{5}$$

By substituting this last expression into equations (1) and (2), one obtains

$$h_{\rm ac} = \frac{8G}{c^4}\frac{\delta m\ R^2\omega^2}{r}\sin^2\theta, \tag{6}$$

and

$$L_{{\rm GW}_{\rm ac}} = -\frac{8}{5}\frac{G}{c^5}\delta m^2 R^4\omega^6\sin^2\theta(16\sin^2\theta + \cos^2\theta). \tag{7}$$

Thereby, we find expressions for the gravitational luminosity and the GW amplitude for the case of a WD accumulating mass, which depends on the accreted mass, the distance to the source, the radius of the star, and how fast it is rotating.

2.2 Magnetic deformation

This section deals with the deformation of the WD induced by its own huge magnetic field. Let us consider that the WD is triaxial, that is, the star has asymmetries with respect to its rotation axis, presenting a triaxial moment of inertia. In order to investigate the effect arising from the magnetic stress on the equilibrium of stars, let us introduce the equatorial ellipticity, defined as (Shapiro & Teukolsky 1983; Maggiore 2008)

$$\epsilon = \frac{I_1 - I_2}{I_3}, \tag{8}$$

where I_1, I_2, and I_3 are main moments of inertia with respect to the (x, y, z) axes, respectively.

If the star rotates around the z-axis, then it will emit monochromatic GWs with a frequency twice the rotation frequency, $f_{\rm rot}$, and amplitude given by (Shapiro & Teukolsky 1983; Maggiore 2008)

$$h_{\rm def} = \frac{16\pi^2 G}{c^4}\frac{I_3 f_{\rm rot}^2}{r}\epsilon, \tag{9}$$

and the rotational energy of the star decreases at a rate given by (Shapiro & Teukolsky 1983; Maggiore 2008)

$$L_{\mathrm{GW_{def}}} = -\frac{32}{5}\frac{G}{c^5} I_3^2 \epsilon^2 \omega_{\mathrm{rot}}^6. \qquad (10)$$

On the other hand, recall that the ellipticity of magnetic origin can also be written as follows (Chandrasekhar & Fermi 1953; Coelho et al. 2014):

$$\epsilon = \frac{35}{24}\frac{B_{\mathrm{s}}^2 R^4}{GM^2}, \qquad (11)$$

where B_{s} is the dipole magnetic field, R and M are the radius and the mass of the star, respectively.

Finally, substituting this last equation into equations (9) and (10), one immediately obtains that

$$h_{\mathrm{def}} = \frac{28\pi^2}{3c^4}\frac{B_{\mathrm{s}}^2 R^6 f_{\mathrm{rot}}^2}{rM}, \qquad (12)$$

and

$$L_{\mathrm{GW_{def}}} = -\frac{98}{45}\frac{B_{\mathrm{s}}^4 R^{12} \omega_{\mathrm{rot}}^6}{c^5 GM^2}. \qquad (13)$$

Note that the two equations just above depend on the rotation frequency and the magnetic field strength.

In contrast, the GWs amplitude can also be written as a function of the variation of the star's rotation frequency $\dot{f}_{\mathrm{rot}}$. In this case, we must consider that the whole spin-down luminosity is converted into GWs. Therewith, we infer an upper limit for amplitude of GWs given by (Aasi et al. 2014)

$$h_{\mathrm{sd}} = \left(\frac{5}{2}\frac{G}{c^3}\frac{I_3 \dot{f}_{\mathrm{rot}}}{r^2 f_{\mathrm{rot}}}\right)^{1/2}. \qquad (14)$$

This equation must be modified to take into account that just a part of the spin-down is due to the GW emission. Thus, we can consider an efficiency, η_{df}, for the variation of the rotation frequency as follows:

$$\dot{\bar{f}}_{\mathrm{rot}} = \eta_{\mathrm{df}} \dot{f}_{\mathrm{rot}}, \qquad (15)$$

such that $\dot{\bar{f}}_{\mathrm{rot}}$ can be interpreted as the part of $\dot{f}_{\mathrm{rot}}$ related to the GW brake. Hence, the GW amplitude can be written as follows:

$$h_{\mathrm{sd}} = \left(\eta_{\mathrm{df}}\frac{5}{2}\frac{G}{c^3}\frac{I_3 \dot{f}_{\mathrm{rot}}}{r^2 f_{\mathrm{rot}}}\right)^{1/2}. \qquad (16)$$

Now, we are ready to calculate the GW amplitude and luminosity for massive fast-spinning WDs. The next section is devoted to this issue as well as the corresponding discussion of the results.

3 RESULTS AND DISCUSSIONS

3.1 Accretion of matter

AE Aqr, is a cataclysmic variable considered peculiar because it has a WD with a very short period of rotation of $P = 33.08$ s, and a high spin-down rate $\dot{P} = 5.64 \times 10^{-14}$ s s^{-1} (see Table 1). This value is considered high because the energy rate needed to explain the WD period variation far exceeds the quiescent luminosity observed in the ultraviolet and X-ray band or even much higher than the bolometric luminosity (Choi & Yi 2000).

Several works proposed different energy emission mechanisms to explain these high spin-down. Eracleous & Horne (1996) proposed a magnetic propeller model, in which the accretion flux of matter by WD is fragmented into discrete amount of mass accumulated and follows the path of the magnetic field lines. However, the fast-rotating magnetic WD moves like a fast-moving propeller expelling much of the matter from the system. Thus, the spin-down energy is consumed to expel the matter. Also, Choi & Yi (2000) proposed as an alternative spin-down mechanism an unconventional configuration for AE Aqr. In this scenario, the WD has a magnetic dipole whose axis is misaligned with the axis of rotation. The source is accreting matter so that the accretion flux follows the field lines to the magnetic poles, and the rapid spin-down is caused by the mountains of accreted matter that produces an asymmetrical deformation in the star structure generating gravitational radiation. This will be the scenario considered in this work to calculate the GW amplitude due to the accretion of matter (see Section 2.1).

Table 1. Parameters of three binary systems containing a fast-rotating WD: period (P), spin-down ($\dot{P}$), adopted WD mass (M), radius (R), and distance from the system to Earth (r).

Systems	P (s)	$\dot{P}$ (10^{-15} s s^{-1})	M (M$_{\odot}$)	R (10^8 cm)	r (pc)
AE Aqr	33.08	56.4	0.80	7.0	100
AR Sco	118.2	392	0.81	7.0	116
RX J0648	13.18	6.0	1.28	3.0	650

Note. (see Patterson 1979; De Jager et al. 1994; Choi & Yi 2000; Mereghetti et al. 2011; Franzon & Schramm 2017).

Here, we apply equation (6) for the system AE Aqr, and extend to AR Sco and RX J0648 in order to calculate the GW amplitude for these objects, considering the scenario of an amount of mass accumulated on the magnetic poles. Moreover, an interesting possibility for the AR Sco is that it may represent an advanced evolutionary stage of the intermediate polar. That is, the magnetic WD may have accreted matter from its companion star by an accretion column, causing it to spin faster and faster until it has reached a very short rotation period (see Table 1).

For these studies, we consider that the angle between the magnetic and rotation axes is $\theta = 30°$. As a result, the GW amplitude reads

$$h_{\mathrm{ac}} = \frac{2G}{c^4}\frac{\delta m R^2 \omega^2}{r}. \qquad (17)$$

The above equation shows that the amplitude depends on the amount of mass accumulated; however, it is not easy to predict how much matter may have been accreted to WD and how much has been dispersed on its surface. Therefore, we assign here four values for the mountain of matter for the three analysed systems: $\delta m = (10^{-3}$, 10^{-4}, 10^{-5}, and 10^{-6} M$_{\odot}$) (see e.g. Lubow & Shu 1975; Welsh et al. 1998; Hellier 2001; Warner 2003, for details about accretion in WDs).

In addition, assuming these values for δm and the parameters listed in Table 1, we obtain h_{ac} for the three systems, which are shown in Fig. 1. It is worth mentioning that for the AR Sco system that has a WD in the mass range of $0.81\ \mathrm{M}_{\odot} < M_{\mathrm{AR}} < 1.29\ \mathrm{M}_{\odot}$ (Franzon & Schramm 2017), we adopted the mass value of 0.81 M$_{\odot}$ to maximize the GW amplitude. For AE Aqr and RX J0648 systems, although mass values are not well established, we use the mass values according to Choi & Yi (2000) and Mereghetti et al. (2011), respectively.

At this point it is interesting to see what kind of information we can obtain from these results. Fig. 2 shows the GW amplitude as a function of the GW frequencies for each δm and the sensitivity curves of the space detectors LISA, BBO, and DECIGO (Yagi & Seto 2011, 2017; Robson et al. 2019). It is worth mentioning that to plot the sensitivity curves, we use the minimum amplitude, h_{min}, that

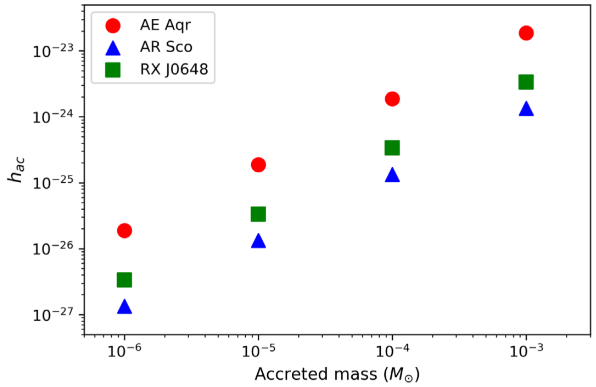

Figure 1. GW amplitude as a function of accreted mass to AE Aqr, AR Sco, and RX J0648.

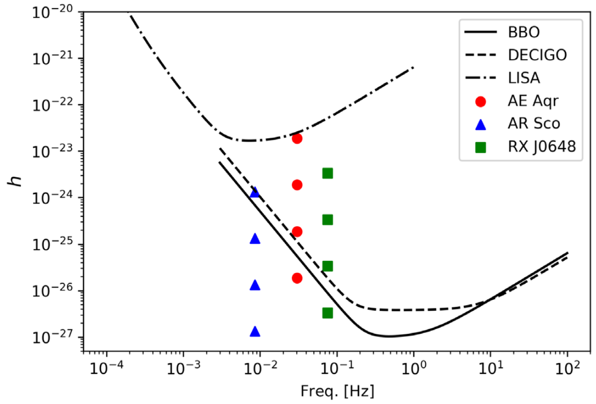

Figure 2. GW amplitude for AE Aqr, AR Sco, and RX J0648 for different values of mass (10^{-3}, 10^{-4}, 10^{-5}, and 10^{-6} M$_\odot$, from top to bottom) and the sensitivity curves for LISA, BBO, and DECIGO for a signal-to-noise ratio SNR = 8 and integration time of $T = 1$ yr.

can be measured by the detector, for a periodic signal, for a given signal-to-noise ratio (SNR) and observation time T (see Maggiore 2008, for more details). Thereby, Fig. 2, as well as Fig. 4, present the GW amplitudes for the sources ($h_{\rm ac}$ and $h_{\rm def}$, respectively) and the sensitivity curves are set to SNR = 8 and $T = 1$ yr.

Notice from Fig. 2 that the systems AE Aqr and RX J0648 emit gravitational radiation with amplitudes that can be detected by BBO and DECIGO as long as $\delta m \geq 10^{-5}$ M$_\odot$. For the AR Sco system, the gravitational radiation emitted by this mechanism would hardly be able to be detected by the three space instruments. This system would need to have a very high mass mountain of $\sim 10^{-3}$ M$_\odot$ to be above, for example, the sensitivity curve of the BBO detector.

Now, we consider the efficiency of this mechanism with respect to the rotational energy rate lost by the systems. First, considering that all spin-down luminosity is converted in GWs, we calculate the amount of mass accumulated, $\delta m_{\rm sd}$, required to explain the loss of rotational energy in each system, namely

$$\delta m_{\rm sd} \approx \left(\frac{5 c^5 L_{\rm sd}}{8 G R^4 \omega^6} \right)^{1/2}. \tag{18}$$

Table 2 shows the values of this parameter for each source. We note that to explain the spin-down luminosity of the star due only to the emission of GWs by the mass accretion mechanism, the WDs should have a large amount of matter at their magnetic poles. The AE Aqr and RX J0648 should have $\delta m \sim 10^{-2}$ and $\delta m \sim 10^{-3}$ M$_\odot$, respectively, while AR Sco should have an even greater δm, around 10^{-1} M$_\odot$. These values of δm are too large, so this GW generation mechanism cannot explain the whole spin-down luminosity.

Table 2. Spin-down luminosity and accumulated mass required to explain the spin-down of WD due to the emission of GWs by the mass accretion mechanism.

Systems	$L_{\rm sd}$ (10^{33} erg s^{-1})	$\delta m_{\rm sd}$ (10^{-2} M$_\odot$)
AE Aqr	19.28	0.98
AR Sco	3.11	17.1
RX J0648	9.53	0.24

Table 3. The efficiency of the generation mechanism of GWs due to the amount of mass accumulated at the WD magnetic poles for different values of δm.

δm (M$_\odot$)	$\eta_{\rm acr}$ ($L_{\rm GW_{acr}}/L_{\rm sd}$)
AE Aquarii	
10^{-3}	1.02×10^{-2}
10^{-4}	1.02×10^{-4}
10^{-5}	1.02×10^{-6}
10^{-6}	1.02×10^{-8}
AR Scorpii	
10^{-3}	3.41×10^{-5}
10^{-4}	3.41×10^{-7}
10^{-5}	3.41×10^{-9}
10^{-6}	3.41×10^{-11}
RX J0648	
10^{-3}	0.175
10^{-4}	1.75×10^{-3}
10^{-5}	1.75×10^{-5}
10^{-6}	1.75×10^{-7}

We now consider the efficiency of the process ($\eta_{\rm acr} = L_{\rm GW_{acr}}/L_{\rm sd}$) for the four δm's considered above, i.e. how much of the spin-down luminosity is converted to gravitational luminosity for every δm (see Table 3). We find that the contribution of gravitational luminosity to the spin-down luminosity is irrelevant, since, for the four values of δm adopted, the efficiency $\eta_{\rm acr} \ll 1$, except for the source RX J0648 with a $\delta m = 10^{-3}$ M$_\odot$ which shows an efficiency of 17.5 per cent (although this value of δm can be considered too great for a WD). Therefore, other mechanisms of energy emission are needed to explain the spin-down of the systems considered here.

3.2 Magnetic deformation

In this section, we consider the generation of GWs due to the deformation of the WD structure of the same binary systems (AE Aqr, AR Sco, and RX J0648) caused by their own intense magnetic field. For this, we use equation (16) to calculate the GW amplitude as a function of the efficiency $\eta_{\rm df} = L_{\rm GW_{def}}/L_{\rm sd}$. The GW amplitudes are shown in Fig. 3 as a function of $\eta_{\rm df}$, where we use the parameters of Table 1 for all three systems.

Fig. 4 shows the strain sensitivities curves for LISA, BBO, and DECIGO for 1 yr of observation time and SNR = 8 and the GW amplitudes as shown in Fig. 3. It is worth noting that AE Aqr and AR Sco are detectable by the LISA detector, only if efficiency $\eta_{\rm df} \geq 10^{-1}$ and $\eta_{\rm df} \geq 10^{-2}$, respectively. On the other hand, notice that all three systems are detectable by BBO and DECIGO as long as AE

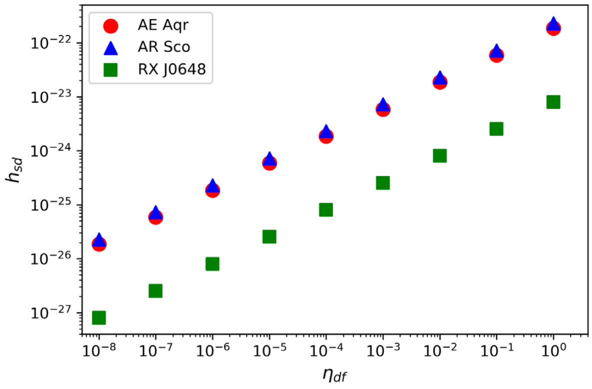

Figure 3. GW amplitude for different values of efficiency ($\eta_{\rm df} = L_{\rm GW_{def}}/L_{\rm sd}$) to AE Aqr, AR Sco, and RX J0648.

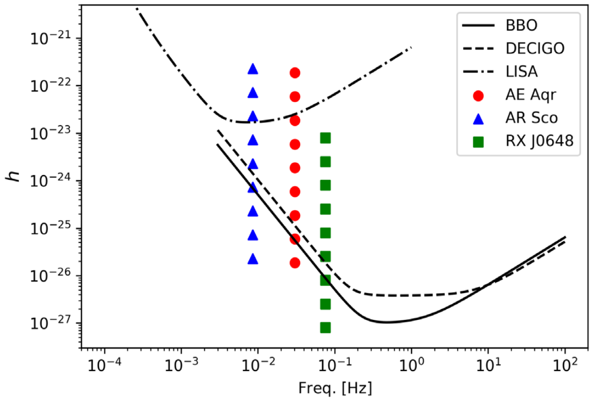

Figure 4. GW amplitudes as presented in Fig. 3 compared to the sensitivity curves of LISA, BBO, and DECIGO for SNR = 8 and integration time of $T = 1$ yr. Here, the efficiency values (1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}, and 10^{-8}) are displayed from top to bottom.

Table 4. Magnetic field strength needed to generate enough gravitational energy to explain all the spin-down luminosity.

Systems	Efficiency $\eta_{\rm df} = 1$ $h_{\rm def}$	B (G)
AE Aqr	1.85×10^{-22}	8.7×10^{10}
AR Sco	2.29×10^{-22}	3.6×10^{11}
RX J0648	8.02×10^{-24}	2.9×10^{11}

Aqr has an efficiency $\eta_{\rm df} \geq 10^{-6}$, AR Sco an efficiency $\eta_{\rm df} \geq 10^{-4}$, and RX J0648 an efficiency $\eta_{\rm df} \geq 10^{-5}$. Thus, even if the GWs have a small contribution to the spin-down of these systems, these sources can emit GWs by the magnetic deformation mechanism with amplitudes that can be detected by the space antennas.

An interesting issue is to calculate the strength of the magnetic field needed to generate sufficient deformation to explain all the spin-down luminosity. Then, using equation (12) we calculate the corresponding GW amplitude (see Table 4). Notice that the magnetic field strength obtained exceed the upper limit established by the canonical model of WD pulsars.

Also, we calculate the magnetic field strength so that these sources can be detected by BBO, which is the most sensitive instrument of the three considered in this study. To do so, we use equation (12) together with the minimum efficiency for which each system is detectable by this instrument. Table 5 shows the values of these magnetic fields along with the amplitude for each system. Notice that the systems must have WDs with high magnetic fields, around (10^9–10^{10}) G, which are about two orders of magnitude larger than the canonical model of WD pulsars.

Table 5. Minimum efficiency for the sources to be measured by the BBO detector along with the amplitude of the GW and the required magnetic field strength.

Systems	Minimum efficiency detected by BBO $\eta_{\rm df}$	$h_{\rm def}$	B (G)
AE Aqr	10^{-6}	1.9×10^{-25}	2.8×10^{9}
AR Sco	10^{-4}	2.3×10^{-24}	3.6×10^{10}
RX J0648	10^{-5}	2.5×10^{-26}	1.6×10^{10}

In addition, we can further calculate the GW amplitude by considering the upper limit values of the magnetic field strength, $B_{\rm dip}$. Thus, knowing that AE Aqr has $B_{\rm dip} = 5.0 \times 10^7$ G (Isakova et al. 2016), AR Sco has $B_{\rm dip} = 5.0 \times 10^8$ G (Buckley et al. 2017), and RX J0648 has $B_{\rm dip} = 1.01 \times 10^8$ G (inferred by the magnetic dipole model), we use equation (11) to calculate the ellipticity of the star, and equation (12) to calculate the GW amplitude. In addition, we compute the gravitational luminosity from equation (13) and the efficiency of this process with respect to the spin-down luminosity.

Table 6 presents the results of this study. Notice that the amplitudes of the GWs shown in this table is very small to be observed by the space detectors, since they are well below their sensitivity curves. Even for 5 yr of integration time, these space instruments will not be able to detect these sources when considering those magnetic field values.

For the three binary systems investigated here, we conclude that from the magnetic deformation mechanism, the WDs require a magnetic field above $\sim 10^9$ G to produce GW amplitudes that can be detectable by BBO, for example. These fields are quite intense, but not unrealistic, since these WDs with surface magnetic fields from 10^6 up to 10^9 G have been confirmed by the recent results of SDSS (Külebi et al. 2009; Kepler et al. 2010, 2013, 2015).

It is worth stressing that although we are interested in HMWDs and most of them have been shown to be massive, it is also important that they be fast-spinning sources, in order to generate GWs in the frequency band where the space antennas are more sensitive. We studied here the three notable sources observed so far that would fit all these conditions (massive, fast-spinning and highly magnetized). Moreover, notice that these three sources are the fastest WDs ever observed.

Evidently, our approach could be applied to new observations of massive, fast-spinning, and highly magnetized WDs.

4 SUMMARY

After the detection of GWs from the merger events, the search for continuous GWs has been of great interest in the scientific community. It is well known that, besides compact binaries, rapidly rotating neutron stars are promising sources of GWs which could be detected in a near future by Advanced LIGO (aLIGO) and Advanced Virgo (AdV), and also by the planned Einstein Telescope (ET). These sources generate continuous GWs whether they are not perfectly symmetric around their rotation axis, i.e. if they present some equatorial ellipticity. Undoubtedly, fast-spinning WDs are also good candidates for this purpose. Here, we investigate the gravitational radiation from these uncommon WDs, which have a high rotation (a few seconds to minutes) and a huge magnetic fields

Table 6. Ellipticity (ϵ), GW amplitude (h_{def}), GW luminosity ($L_{\mathrm{GW_{def}}}$), and efficiency of the mechanism (η_{df}) for the upper limit of magnetic field (B_{dip}) of each system.

Systems	B_{dip} (G)	ϵ	h_{def}	$L_{\mathrm{GW_{def}}}$ (erg s^{-1})	η_{df}
AE Aqr	5.0×10^{7}	5.1×10^{-9}	6.2×10^{-29}	2.13×10^{21}	1.1×10^{-13}
AR Sco	5.0×10^{8}	5.3×10^{-7}	4.6×10^{-28}	1.25×10^{22}	4.02×10^{-12}
RX J0648	1.0×10^{8}	2.8×10^{-10}	9.5×10^{-31}	1.33×10^{20}	1.4×10^{-14}

(10^{6} to $\sim 10^{10}$ G), using two emission mechanisms: matter accretion and magnetic deformation. These WDs usually have a high spin-down rate that is not fully explained by the electromagnetic counterpart.

Then, we study the following three binary systems: AE Aqr, AR Sco, and RX J0648. First, we consider the role of the aforementioned deformation due to the accretion of matter in the putative generation of GWs by the HMWDs. Our calculations show that the AE Aqr and RX J0648 systems are good candidates for BBO and DECIGO if they have an amount of mass accumulated of $\delta m \geq 10^{-5}$ M$_{\odot}$, for 1 yr of integration time. AR Sco, on the other hand, is unlikely to be detected because it requires a very large amount of mass accumulated in the magnetic pole of the WD.

Secondly, regarding the magnetic deformation mechanism, we note that the three binary systems studied require that the WD has a magnetic field above $\sim 10^{9}$ G to emit gravitational radiation with amplitudes that are detectable by BBO, for example. However, these WDs are inferred to have magnetic fields with intensity around two orders of magnitude smaller.

In addition, it is worth stressing that the efficiency of both mechanisms ($\eta = L_{\mathrm{GW}}/L_{\mathrm{sd}}$) is very small when considering the three binary systems studied. Thus, gravitational radiation has an irrelevant contribution to the spin-down luminosity of these systems.

ACKNOWLEDGEMENTS

MFS thanks CAPES for the financial support. JGC is likewise grateful to the support of CNPq (421265/2018-3 and 305369/2018-0). JCNA thanks FAPESP (2013/26258-4) and CNPq (307217/2016-7) for partial financial support. Last but not least, we thank the referee for the suggestions and criticisms that helped to improve the manuscript.

REFERENCES

Aasi J. et al., 2014, ApJ, 785, 119
Abbott B. P. et al., 2016, Phys. Rev. Lett., 116, 061102
Abbott B. P. et al., 2017a, Phys. Rev. Lett., 118, 221101
Abbott B. P. et al., 2017b, Phys. Rev. Lett., 119, 141101
Abbott B. P. et al., 2017c, Phys. Rev. Lett., 119, 161101
Abbott B. P. et al., 2017d, ApJ, 851, L35
Abbott B. P. et al., 2019, ApJ, 882, L24
Althaus L. G., García-Berro E. , Isern J., Córsico A. H., 2005, A&A, 441, 689
Althaus L. G., García-Berro E., Isern J., Córsico A. H., Rohrmann R. D., 2007, A&A, 465, 249
Amaro-Seoane P. et al., 2017, preprint (arXiv:1702.00786)
Barstow M. A., Jordan S., O'Donoghue D., Burleigh M. R., Napiwotzki R., Harrop-Allin M. K., 1995, MNRAS, 277, 971
Bonazzola S., Gourgoulhon E., 1996, A&A, 312, 675
Buckley D. A. H., Meintjes P. J., Potter S. B., Marsh T. R., Gänsicke B. T., 2017, Nat. Astron., 1, 29
Camisassa M. E. et al., 2019, A&A, 625, A87
Castanheira B. G., Kepler S. O., Kleinman S. J., Nitta A., Fraga L., 2013, MNRAS, 430, 50
Chandrasekhar S., Fermi E., 1953, ApJ, 118, 116
Choi C.-S., Yi I., 2000, ApJ, 538, 862
Coelho J. G., Marinho R. M., Malheiro M., Negreiros R., Cáceres D. L., Rueda J. A., Ruffini R., 2014, ApJ, 794, 86
Curd B., Gianninas A., Bell K. J., Kilic M., Romero A. D., Allende Prieto C., Winget D. E., Winget K. I., 2017, MNRAS, 468, 239
De Araujo J. C. N., Coelho J. G., Costa C. A., 2016a, ApJ, 831, 35
De Araujo J. C. N., Coelho J. G., Costa C. A., 2016b, J. Cosmol. Astropart. Phys., 2016, 023
De Araujo J. C. N., Coelho J. G., Costa C. A., 2017, Eur. Phys. J. C., 77, 350
De Araujo J. C. N., Coelho J. G., Ladislau S. M., Costa C. A., 2019, preprint (arXiv:1906.00774)
De Jager O., Meintjes P., O'Donoghue D., Robinson E., 1994, MNRAS, 267, 577
Eracleous M., Horne K., 1996, ApJ, 471, 427
Franzon B., Schramm S., 2017, MNRAS, 467, 4484
Gao H., Cao Z., Zhang B., 2017, ApJ, 844, 112
Gentile Fusillo N. P. et al., 2018, MNRAS, 482, 4570
Harry G. M., Fritschel P., Shaddock D. A., Folkner W., Phinney E. S., 2006, Class. Quantum Gravity, 23, 4887
Hellier C., 2001, Cataclysmic Variable Stars. Springer, Berlin
Hermes J. J., Kepler S. O., Castanheira B. G., Gianninas A., Winget D. E., Montgomery M. H., Brown W. R., Harrold S. T., 2013, ApJ, 771, L2
Isakova P. B., Ikhsanov N. R., Zhilkin A. G., Bisikalo D. V., Beskrovnaya N. G., 2016, Astron. Rep., 60, 498
Jiménez-Esteban F. M., Torres S., Rebassa-Mansergas A., Skorobogatov G., Solano E., Cantero C., Rodrigo C., 2018, MNRAS, 480, 4505
Kalita S., Mukhopadhyay B., 2019, MNRAS, 490, 2692
Kawamura S. et al., 2006, Class. Quantum Gravity, 23, S125
Kepler S. O., Kleinman S. J., Pelisoli I., Peçanha V., Diaz M., Koester D., Castanheira B. G., Nitta A., 2010, in Werner K., Rauch T., eds, AIP Conf. Proc. Vol. 1273, Magnetic White Dwarfs in the SDSS and Estimating the Mean Mass of Normal DA and DB WDs, Am. Inst. Phys., New York, p. 19
Kepler S. O. et al., 2013, MNRAS, 429, 2934
Kepler S. O. et al., 2015, MNRAS, 446, 4078
Külebi B., Jordan S., Euchner F., Gänsicke B. T., Hirsch H., 2009, A&A, 506, 1341
Külebi B., Jordan S., Nelan E., Bastian U., Altmann M., 2010, A&A, 524, A36
Liebert J., Schmidt G. D., Green R. F., Stockman H. S., McGraw J. T., 1983, ApJ, 264, 262
Lubow S. H., Shu F. H., 1975, ApJ, 198, 383
Maggiore M., 2008, Gravitational Waves: Vol. 1: Theory And Experiments. Oxford Univ. Press, Oxford
Marsh T. R. et al., 2016, Nature, 537, 374
Mereghetti S., Tiengo A., Esposito P., La Palombara N., Israel G. L., Stella L., 2009, Science, 325, 1222
Mereghetti S., La Palombara N., Tiengo A., Pizzolato F., Esposito P., Woudt P. A., Israel G. L., Stella L., 2011, ApJ, 737, 51
Mukhopadhyay B., Rao A. R., Bhatia T. S., 2017, MNRAS, 472, 3564
Patterson J., 1979, ApJ, 234, 978
Pereira J. P., Coelho J. G., de Lima R. C. R., 2018, Eur. Phys. J. C., 78, 361

Robson T., Cornish N. J., Liug C., 2019, Class. Quantum Gravity, 36, 105011

Schmidt G. D., West S. C., Liebert J., Green R. F., Stockman H. S., 1986, ApJ, 309, 218

Schmidt G. D., Bergeron P., Liebert J., Saffer R. A., 1992, ApJ, 394, 603

Shapiro S. L., Teukolsky S. A., 1983, Black Holes, White Dwarfs and Neutron Stars: The Physics of Compact Objects. John Wiley & Sons, New York

Terada Y. et al., 2008, PASJ, 60, 387

Warner B., 2003, Cataclysmic Variable Stars. Cambridge Astrophysics, Cambridge

Welsh W. F., Horne K., Gomer R., 1998, MNRAS, 298, 285

Yagi K., Seto N., 2011, Phys. Rev. D, 83, 044011

Yagi K., Seto N., 2017, Phys. Rev. D, 95, 109901

This paper has been typeset from a TEX/LATEX file prepared by the author.