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TRAJECTORY ANALYSIS OF A SPACECRAFT MAKING A THREE-DIMENSIONAL POWERED SWING-BY MANEUVER

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The present paper studies the problem of orbital maneuvers performed combining the passage of a spacecraft by a celestial body with an impulse applied to the spacecraft during the close approach. The motion of the spacecraft is assumed to be in the three-dimensional space, thus allowing plane change. This type of maneuver can also send the spacecraft to a point far from the orbit or to end in capture or collision of the spacecraft by the celestial body. The effects of the variations in the orbital plane, energy and the angular momentum of the spacecraft are presented.

INTRODUCTION

Swing-By maneuvers are used in space missions to send a spacecraft to targets like planets, moons or asteroids. This technique is based in making a close approach with a celestial body, which gives or removes energy from the spacecraft. It can also be used to generate the capture or escape of a spacecraft relative to the celestial body. When a spacecraft passes near a celestial body and uses the gravity of this body to change its orbit, we call it "pure gravity Swing-By maneuver." This change includes the modification of the velocity, energy and angular momentum of the spacecraft. This is a type of maneuver well known in the literature, since it was already used in several space missions. The goal is usually fuel economy, considering that the Swing-By is equivalent to the application of an impulse with zero fuel expenditure[1-9].

Several works were developed considering the motion of the spacecraft limited to the plane of the primaries, combining the "pure Swing maneuver" with an impulse applied in the space vehicle at some point of the trajectory, in order to optimize the maneuver. Called "powered Swing-By", the use of this maneuver is interesting when the energy obtained from the pure gravity Swing-By maneuver is not enough to meet the needs of the mission[10-14]. Other papers present a study for the powered maneuver considering the primary bodies in elliptical orbits, which is a more realistic maneuver in several systems of primaries[15-18].

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To expand this study to the three-dimensional space, we will use the work of Prado (2000)[19], which presented analytical equations for a Swing-By maneuver in the three-dimensional space for systems with circular orbits and without the application of an impulse. Formiga and Santos (2015)[20] and Prado and Felipe (2007)[21] also performed analytical studies for systems with circular orbits and an impulsive maneuver applied in the periapsis of the orbit of the spacecraft. Other studies related to the three-dimensional Swing-By are also available[22-24]. Recently, Gagg Filho and Fernandes[25] formulated an orbital transfer problem for an Earth-to-Earth mission between non-coplanar orbits considering the occurrence of a lunar Swing-By during the transfer orbit, with the goal of making an inclination change.

The focus of the present work is the analysis of the energy and inclination variations in the orbit of a spacecraft during a powered Swing-By maneuver, with the impulse applied in the periapsis of the orbit, considering the three-dimensional situation. In all the situations considered in the present paper, numerical simulations are made, since the restricted three-body problem is used and it does not have analytical solutions.

DYNAMICAL SYSTEM

The restricted circular three-body problem (RCTBP)[26] is used in the present paper. It has M_1 as the body with the largest mass $(1-\mu)$ and M_2 as the secondary body of the system, where μ is the mass parameter of the system, which is the mass of the secondary body divided by the total mass of the system. Both primaries are assumed to be in circular orbits around their common center of mass. M_3 represents the spacecraft, with negligible mass, which is moving in the three-dimensional space.

The impulse referred to the powered part of the maneuver is applied at the time of the closest approach of the spacecraft with M_2. Its magnitude and direction are free parameters. In the nomenclature used here, δV is the magnitude of the impulse, ω is the angle, in the plane $V_x - V_y$, that defines the direction of the impulse and η is an out-of-plane angle that completes the description of the direction of the impulse. Both angles are measured in the V_x, V_y, V_z frame, originating at the periapsis and parallel to x, y, z. The position vector at the periapsis of the orbit ($\vec{r}_p$) is defined by α, the angle in the $x-y$ plane; β, an out-of-plane angle $(-90° < \beta < 90°)$ and r_p, its magnitude that is measured from the center of M_2 to the point P (see Figure 1). The velocity vector at periapsis ($\vec{V}_p$) is given by γ (the angle between $\vec{V}_p$ and the horizontal plane that passes by the periapsis) and the magnitude V_p.

The equations of motion of the spacecraft are given by: $\ddot{x} - 2\dot{y} = \Omega_x, \ddot{y} + 2\dot{x} = \Omega_y$ and $\ddot{z} + z = \Omega_z$, where Ω_x, Ω_y and Ω_z are the partial derivatives of $\Omega = \frac{1}{2}(x^2+y^2+z^2) + \frac{1}{2}\mu(1-\mu) + \frac{(1-\mu)}{r_1} + \frac{\mu}{r_2}$, with $r_1 = \sqrt{(x+\mu)^2+y^2+z^2}$ and $r_2 = \sqrt{(x-1+\mu)^2+y^2+z^2}$. They were numerically integrated to find the trajectories.

Equations 1 and 2 give the position and velocity of the spacecraft at the time of the impulsive maneuver, in the periapsis of its orbit around M_1, in the rotating frame. If $\delta V = 0$, there is only the natural motion of the spacecraft around the secondary body, which is the pure gravity Swing-By.

$$
\begin{gathered}
x = (1-\mu) + r_p \cos\beta \cos\alpha \\
y = r_p \cos\beta \sin\alpha \\
z = r_p \sin\beta
\end{gathered}
\tag{1}
$$

$$
\begin{aligned}
\dot{x} &= -V_p \sin\gamma \sin\beta \cos\alpha - V_p \cos\gamma \sin\alpha + r_p \cos\beta \sin\alpha \; + \delta V \cos\eta \cos\omega \\
\dot{y} &= -V_p \sin\gamma \sin\beta \, sen\,\alpha - V_p \cos\gamma \cos\alpha - r_p \cos\beta \cos\alpha + \delta V \cos\eta \sin\omega \\
\dot{z} &= V_p \cos\beta \sin\gamma + \delta V \sin\eta
\end{aligned}
\tag{2}
$$

Figure 1 shows that the spacecraft leaves point A, a point where the influence of the secondary body in the spacecraft can be neglected, passes by the point P, the periapsis of the trajectory of the spacecraft around M_2, where the impulse is applied to change the trajectory instantly, and then it goes to point B, also a point where the effect of M_2 on the spacecraft can be neglected. The dashed red line represents the first orbit, before the impulse. The continuous red line is the new trajectory, after the impulse. The velocity of the spacecraft with respect to M_1 is obtained and used to get the two-body energy spacecraft-M_1 and the angular momentum with respect to M_1, as well.

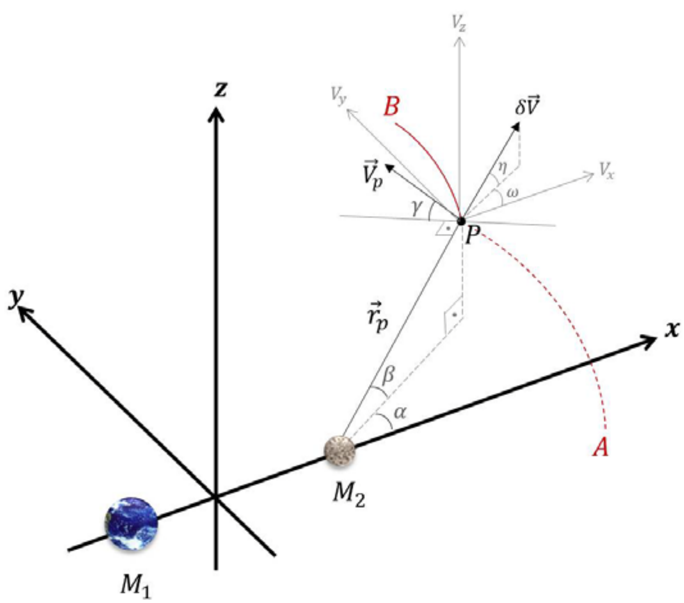


Figure 1. Three-dimensional powered Swing-By maneuver.

The energy variation (ΔE) is calculated by the difference between the energy at the points A and B. If this difference is negative, the combination of the gravitational and impulsive part of the maneuver decelerate the spacecraft, reducing its velocity and, consequently, its energy. This is a characteristic that favors the capture or collision of the spacecraft by the secondary body.

Another feature that helps captures and collisions is the geometry of the application of the impulse. If there is a component in the direction of the secondary body, the spacecraft tends to go towards the body, therefore facilitating these occurrences. If there is a component in the opposite direction, the spacecraft moves away from the body, so minimizing the effect of its gravity.

From the angular momentum, it is possible to calculate the inclination of the orbit of the spacecraft with respect to the plane of the lunar orbit, in the points A and B. Therefore, the variation of the inclination is given by $\Delta i = i_B - i_A = \cos^{-1}\left(\frac{Cz_B}{|C|}\right) - \cos^{-1}\left(\frac{Cz_A}{|C|}\right)$, where Cz_A and Cz_B are the z-components of the angular momentum at the points A and B, respectively, and $|C|$ is the magnitude of the angular momentum. If we consider β, γ and η equal to zero, we have the two-dimensional version of the problem. In this case the inclination is always zero and the spacecraft moves in the plane of the primaries ($x - y$).

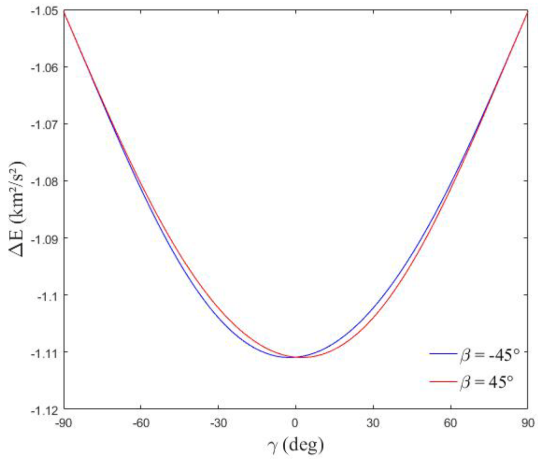
To perform some numerical simulations, maneuvers around the Moon in the Earth-Moon system will be used. Therefore, the mass parameter of M_2 is $\mu = 0.01214$, 1.0 Moon's radius is equal to 1730 km and 1.02 km/s is the velocity of the Moon around the Earth. The velocity of the spacecraft at the periapsis of its orbit is 2.47 km/s, equivalent to a velocity of approach of 1.0 km/s.

Figure 2 shows the effect of the out-of-plane component of the velocity at the periapsis in the variation of energy and inclination in a pure gravity maneuver, without impulse. The goal is just to give an idea of the effects of this angle, but, since it depends on the approach trajectory, its values are typically near zero. Most of the trajectories will be coming in a plane near the orbital plane of the primaries, which gives a zero value for this variable. According to Felipe and Prado (2000)[27] the effects of γ depend on the initial conditions of the trajectory.

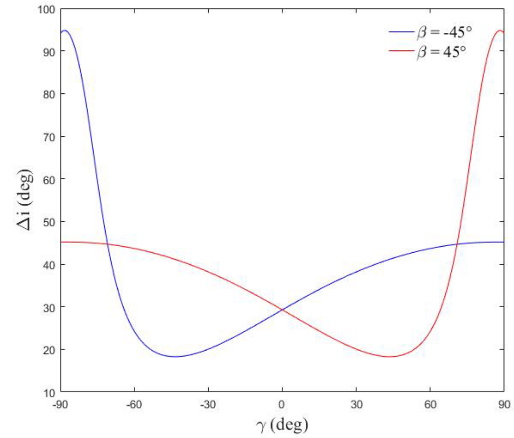
It is clear the symmetry of γ in the energy and inclination variations, for α equal to 90° and 270°. In Figure 2(a) the energy variations are negative and in Figure 2(c) they are positive, but both with the same magnitude. The curves are almost coincident for $\beta = -45°$ and $\beta = 45°$. In Figures 2(b) and 2(d) the inclination also has the same magnitude, but with opposite signs. In this case $\beta = -45°$ and $\beta = 45°$ are also opposite to γ. When $\gamma = 0°$, the highest energy losses occur after the close encounter for $\alpha = 90°$ and the highest gains for $\alpha = 270°$. This is also the condition, for both cases, where the maneuver has the same Δi. As expected, γ has a much stronger influence in the variation of inclination, compared to its effects in the variation of energy. The largest variation of energy occurs when $\gamma = 0°$, a planar approach. The largest variation in inclination occurs when $\gamma = \pm 90°$.

Regarding the effects of the periapsis geometry, a color map of the energy and inclination variation for a maneuver with $r_p = 1.1$ Moon's radius, $\gamma = 0°$ and zero impulse is presented, showing the effects of gravity alone.

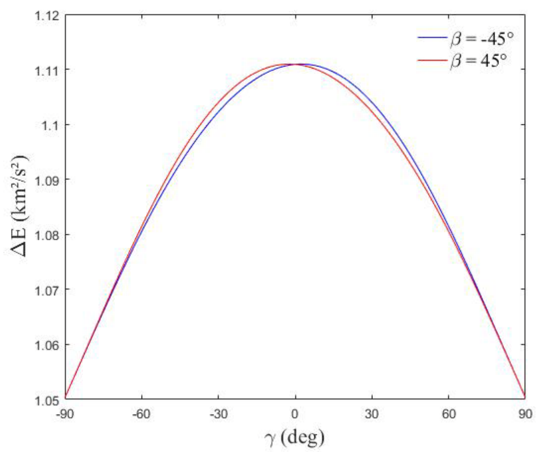
From the gravitational part of the maneuver (no impulse applied to the spacecraft) we see that the energy and inclination are symmetrical with respect to the line $\alpha = 180°$. Since the spacecraft gains energy after the close encounter, for $180° < \alpha < 360°$, its inclination decreases. The opposite occurs if $0° < \alpha < 180$, and the inclination increases. According to Broucke (1988)[28], gravity removes energy (with a maximum when $\alpha = 90°$) when the spacecraft is passing in front of the secondary body. It gets a maximum gain of energy when $\alpha = 270°$, with the spacecraft passing behind the secondary body. The maximum effects in the energy variation occur when $\beta = 0°$, a planar maneuver. This planar maneuver allows variations in the inclination only of ±180°.



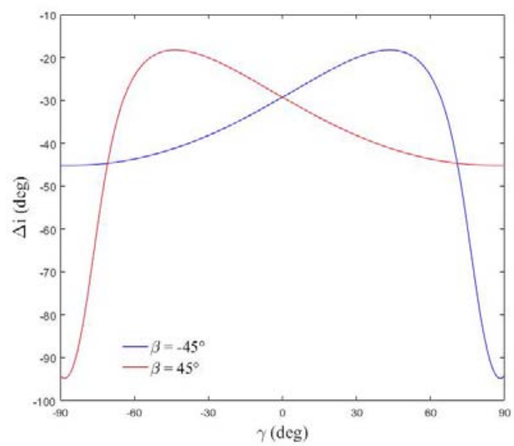
a) Energy variation, for $\alpha = 90°$.



b) Inclination variation, for $\alpha = 90°$.

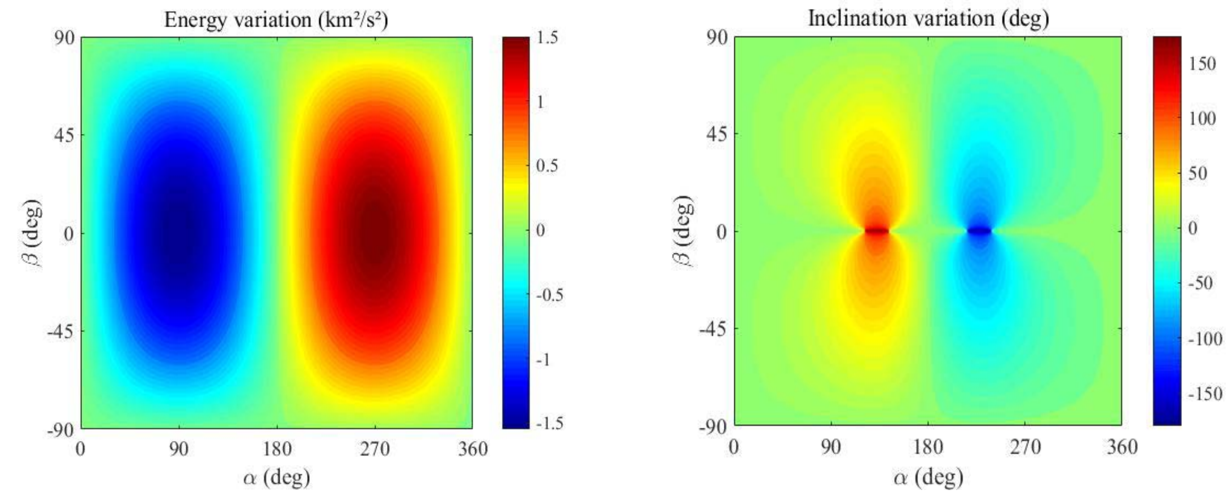


c) Energy variation, for $\alpha = 270°$.



d) Inclination variation, for $\alpha = 270°$.

Figure 2. Effect of γ in the energy and inclination variations.



(a) (b)

Figure 3. Energy and inclination variation of the spacecraft.

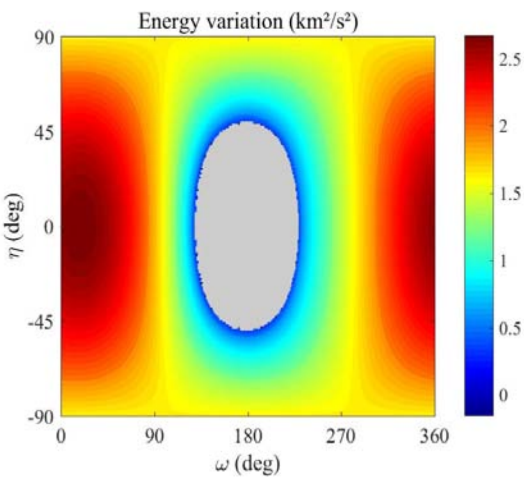
It is important to emphasize that the application of the impulse makes significant changes in the trajectory of the spacecraft, when compared with the situation where only the gravitational part of the maneuver is acting. For the numerical simulations, the focus is in the energy gains, so it is always used $\alpha = 270°$. The direction of the impulse is a free parameter, varying in the intervals $0° < \omega < 360°$ and $-90° < \eta < 90°$. The magnitude $\delta V = 0.4$ km/s was adopted. An analysis of the energy variation and inclination variation resulting from these combinations are studied.

ENERGY VARIATION

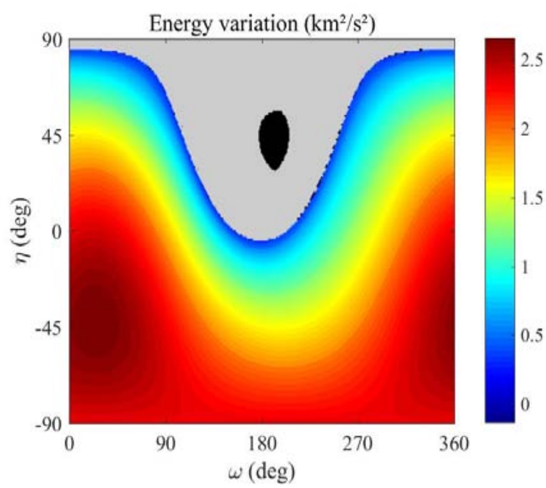
The first part of the present study will look at the energy variations. For the powered maneuvers, these variations come from a combination of the direct energy given by the impulse applied and the new variations due to the modification of the trajectory of the spacecraft, which changes the energy variation due to the gravity part of the maneuver. The impulse may have a component in the direction of M_2, causing the spacecraft to approach M_2 and then to intensify the gravity part of the maneuver; or a component against M_2, pushing the spacecraft away and so decreasing the effect of the gravity part of the maneuver. These characteristics make the powered Swing-By a complex study, but valid when the pure maneuver is not enough to meet the needs of the mission.

Figure 4 shows the energy variation when the periapsis is in the plane of the primaries, in the situation of maximum gains of energy ($\alpha = 270°$ and $\beta = 0°$). The results are showed for three different periapsis velocity inclination: $\gamma = 0°, -45°,$ and $\gamma = 45°$. The goal is to get some initial information about the behavior of the system, in particular the effects of γ. The first observation is the presence of captures and collisions regions. The black regions represent the captures. For this condition the spacecraft remained around the Moon until the final integration time. The gray regions represent the collisions, when the spacecraft hit the surface of the Moon. Figure 4 confirms and quantifies the expected result that the maximum variation of energy occurs when the impulse is applied in the direction of motion. It also shows the occurrence of collisions around the point ($\omega = 180°, \eta = 0°$). It happens because this point represents a maneuver where the impulse is applied opposite to the motion of the spacecraft, so reducing its velocity and helping collisions. In Fig. 4(a) the maximum variation of energy occurs at $\eta = \omega = 0°$, because the spacecraft is approaching M_2 from a planar maneuver ($\gamma = 0° = \beta$). Note also that, for $0° < \omega < 90°$, the region of largest energy gains (dark red) is wider compared to the region $270° < \omega < 360°$. Although both intervals have an impulse component in the direction of the motion of the spacecraft, ω between 0° and 90° gives the other component in the direction of the secondary body, maximizing the effect of gravity. For $270° < \omega < 360°$, the impulse sends the spacecraft away from the secondary body. The cases with negative variations of energy are in the limit line for the collisions. In Fig. 4(b) this maximum happens at $\eta = -45°$ and $\omega = 0°$, because the spacecraft is approaching M_2 from a plane that is -45° inclined with the orbital plane of the primaries. The region of collisions is shifted accordingly, and a region of captures appear. Figure 4(c) is symmetric to Fig. 4(b), because the spacecraft is approaching M_2 from a plane that is 45° inclined with the orbital plane of the primaries.

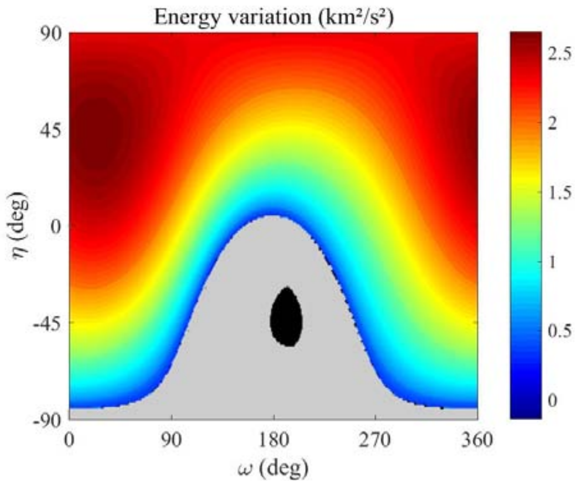
Regarding the magnitude of the energy variations, in Figure 4(a) they are in the range -0.13 to 2.7 km²/s², which means that even negative numbers are possible, where the impulse is acting to remove energy from the spacecraft even in a maneuver with an angle of approach of 270°. The same happens in Figs. 4(b) and 4(c), where the interval is approximately from -0.15 to 2.72 km²/s².



a) $\gamma = 0°$



b) $\gamma = -45°$



c) $\gamma = 45°$

Figure 4. Energy variation for $\alpha = 270°$ and $\beta = 0°$.

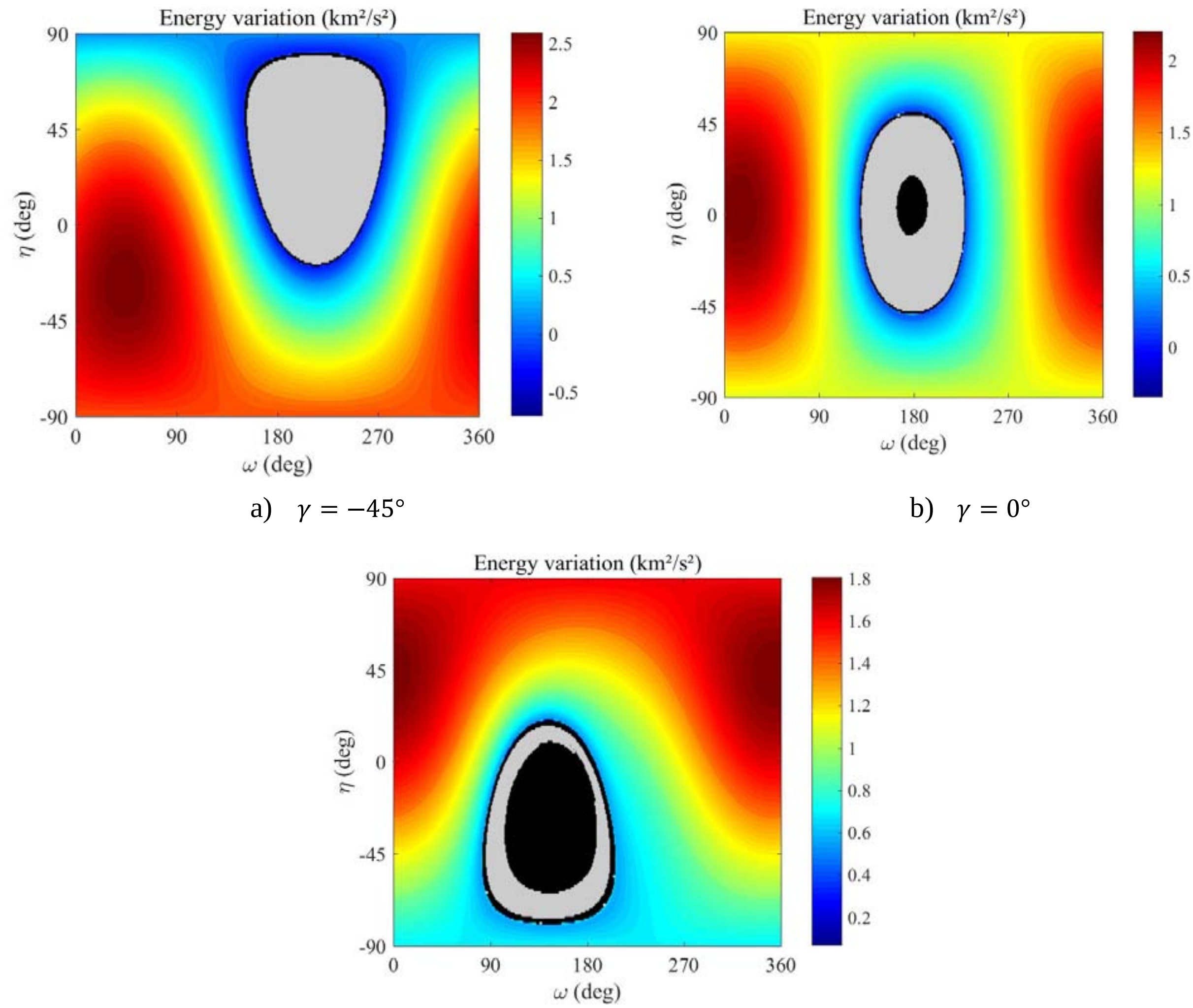
Next, the energy variations for $\alpha = 270°$ and $\beta = -45°$ is presented in Figure 5. We can see that the conditions for the gains and losses of energy are similar to those shown in Figure 4, but the magnitude of ΔE and the conditions of captures and collisions have changed.

In Figure 5(a) ($\gamma = -45°$) the magnitude of ΔE are in the interval -0.7 to 2.65 km²/s². There are significantly more collisions, which region is limited by the captures and then by the region of energy losses. The maximum ΔE occurs for $\omega = 44°$ and $\eta = -27°$ and the minimum for $\omega = 226°$ and $\eta = -18°$.

In Figure 5(b) ($\gamma = 0°$) the energy variations are in the interval -0.34 to 2.25 km²/s², with a maximum at $\omega = 15°$ and $\eta = 3°$. In these conditions the impulse is applied making a small angle with respect to the plane of the primaries with components in the direction of the motion of the spacecraft and with another one in the direction of the secondary body, so maximizing the effect of gravity. In addition to the numerous cases of collisions, a capture region appears in the center of the map, for ω around 180° and η near zero.

When $\gamma = 45°$ (Figure 5(c), the maximum energy variation is 1.83 km²/s² an it occurs for $\omega = 355°$ and $\eta = 43°$.The minimum is 0.06 km²/s², for $\omega = 104°$ and $\eta = -75°$. The conditions of the impulse that result in captures (black region) and collisions (gray region) are approximately $90 < \omega < 180°$ and $-75° < \eta < 0°$. It means that the impulse has one component decelerating the spacecraft and another one sending it to M_2, therefore helping the occurrence of these cases.

A situation where $\alpha = 270°$ and $\beta = 45°$ was simulated, and the results are symmetric with respect to the line $\eta = 0°$, when compared to the situation with $\alpha = 270°$ and $\beta = -45°$, shown in Figure 5. So, the figures are omitted.

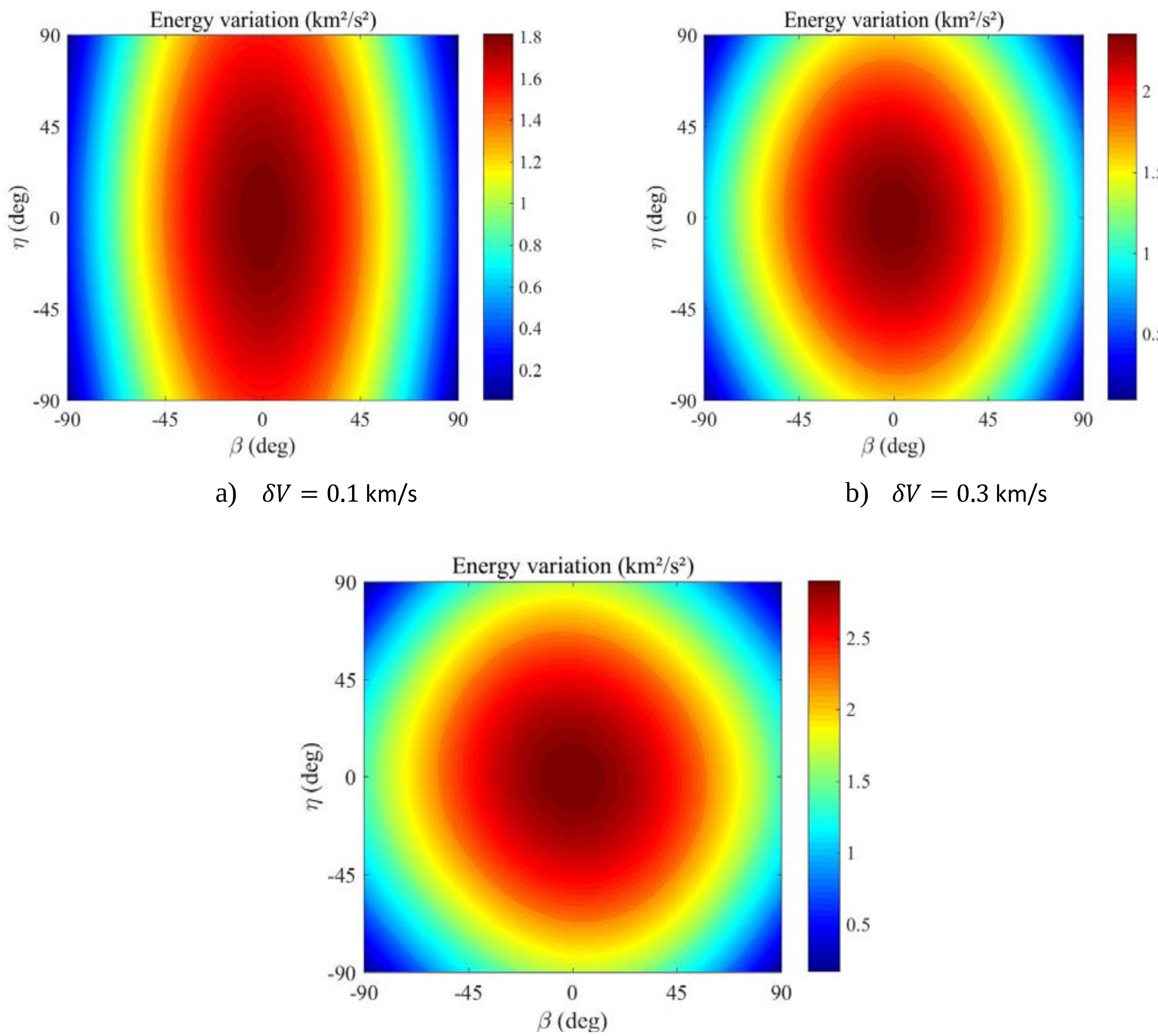


a) $\gamma = -45°$ b) $\gamma = 0°$

c) $\gamma = 45°$

Figure 5. Energy variation for $\boldsymbol{\alpha = 270°}$ and $\boldsymbol{\beta = -45°}$.

The next point is to study the energy variation as a function of the parameters β and η. A typical Swing-By maneuver with the Moon will have a trajectory coming from an orbit around the Earth that is nearly coplanar with the orbit of the Moon, to avoid expensive out-of-plane maneuvers. In this way, the best parameters to control the maneuver to get the desired balance among variations of energy and inclination is β, because it is easy to make very small mid-course corrections to reach the Moon above or below the equator, and η, the out-of-plane direction of the impulse. The same is true for any system of primaries. Figure 6 shows the energy variation for $\alpha = 270°$ and $\omega = 0°$. This is the situation of maximum variation of energy ($\alpha = 270°$) with the impulse applied in the direction of motion of the spacecraft ($\omega = 0°$). The first point to be noted is that the maximum variation of energy occurs at the point (0,0), which means a planar maneuver with no out-of-plane component for the impulse. This result is expected, but the quantification of the variations as a function of these parameters are interesting and important results. It is also measured the increase of the variation of energy when the values of the magnitude of the impulse are larger, which can be noted by the scale of the color codes for the three situations. The general shapes of the plots also show an important point. The first plot ($\delta V = 0.1$ km/s) has borders between the colors closer to verticals, because the out-of-plane location of the Swing-By (β) are stronger than the effects of the out-of-plane component of the impulse (η). It happens because the magnitude of the impulse is small. Increasing this magnitude to $\delta V = 0.3$ km/s those borders become more curved, indicating the increasing effects of the propulsive part of the maneuver. For the last plot, for $\delta V = 0.5$ km/s, those borders are circular, which means that the effects of both variables are of the same order of magnitude. A very large impulse would make those borders to be closer to horizontals.



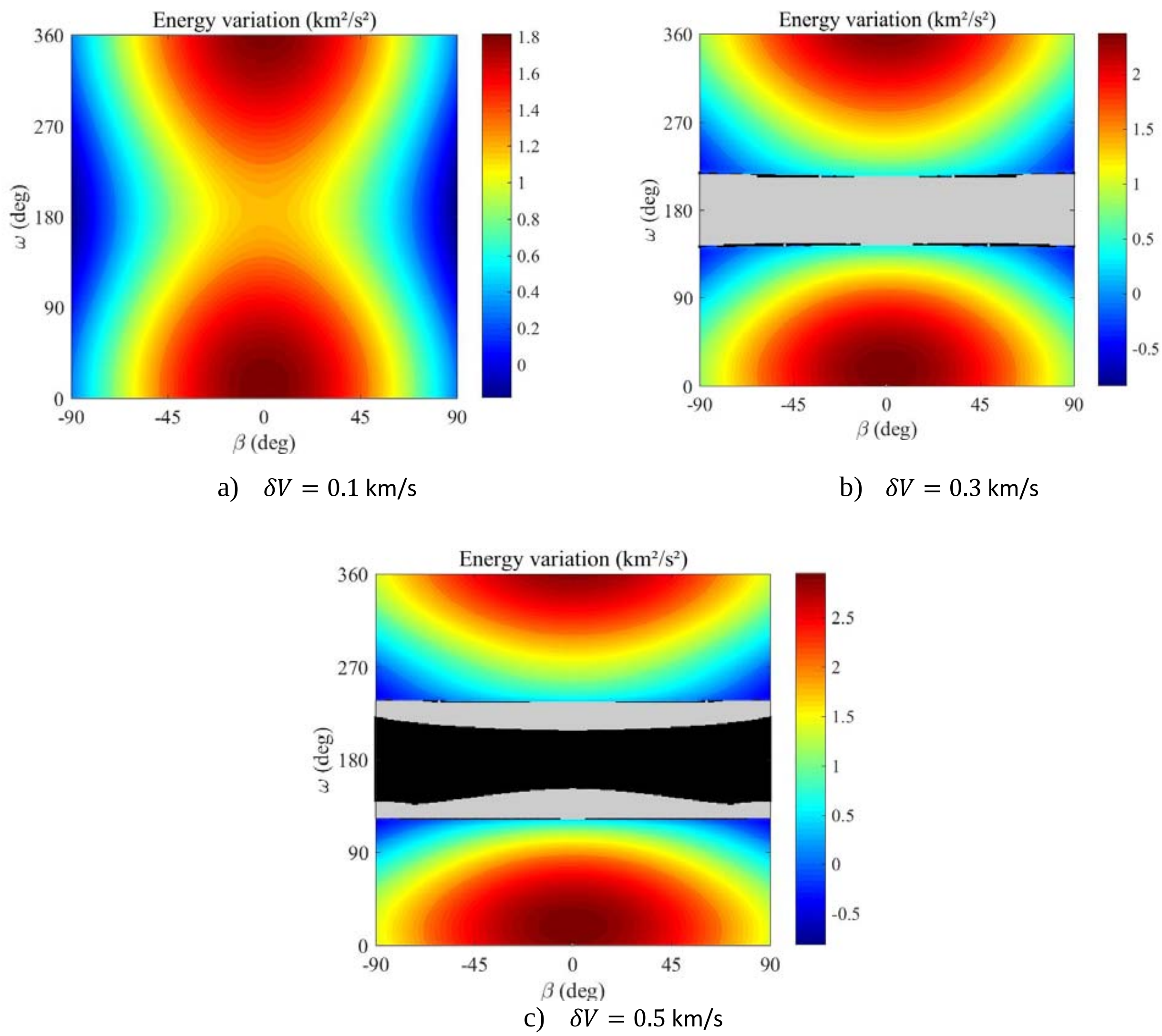
a) $\delta V = 0.1$ km/s

b) $\delta V = 0.3$ km/s

c) $\delta V = 0.5$ km/s

Figure 6. Energy variation for $\alpha = 270°$ and $\omega = 0°$.

Next, we made a study considering the effects of the variables β and ω. Figure 7 shows the results, which are the energy variations for $\alpha = 270°$ (maximum gains of energy and $\eta = 0°$ (planar impulse). It is observed that the maximum variations of energy occur at the points (0,0) and (0,360°), which means a planar maneuver with the impulse applied in the direction of the motion of the spacecraft. Of course, 0 and 360° represent the same direction. This result is also expected, but it is accurately quantified here as a function of the variables involved. It is also showed the increase in the gains of energy due to larger magnitudes of the impulse, by looking at the color codes for the three magnitudes of impulse showed in Figure 7. The general shapes of the plots are also different. The first results made for an impulse with magnitude $\delta V = 0.1$ km/s shows borders between colors that are closer to verticals, as observed in Figure 6. It happens for the same reasons already explained. The out-of-plane position of the Swing-By (β) gives stronger effects in the maneuver compared to the direction of (ω), since the magnitude of the impulse is small. The next results, for $\delta V = 0.3$ km/s, have opposite results and those borders are curved, but closer to horizontals. It happens because the effects of the direction of the propulsive part of the maneuver are now more important than the location of the periapsis of the incoming orbit. The physical phenomenon behind those facts is that giving larger impulses in the region of maximum variations of energy gives larger gains, but giving larger impulses in the regions of minimum variations of energy reduces even more these values. The last plot, for $\delta V = 0.5$ km/s, also has borders closer to horizontals, but now a region of captures and escapes appear. This is a consequence of the increase of the impulse, which now is able to reduce the velocity of the spacecraft to allow those captures and escapes to occur. Variations of energy are maximum for $\beta = 0°$, as expected[19].



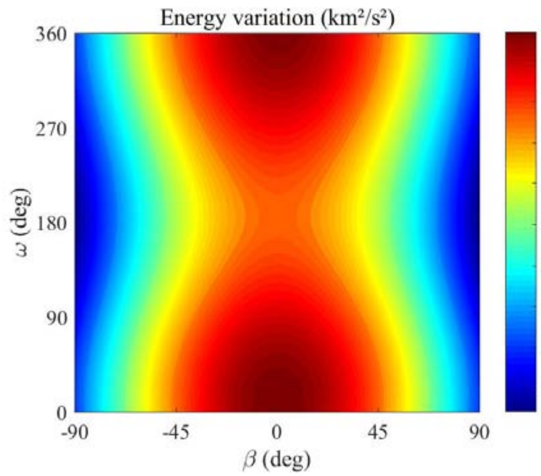
a) $\delta V = 0.1$ km/s

b) $\delta V = 0.3$ km/s

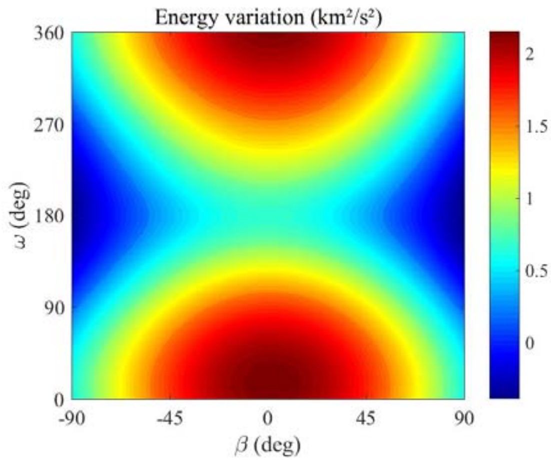
c) $\delta V = 0.5$ km/s

Figure 7. Energy variation for $\alpha = 270°$ and $\eta = 0°$.

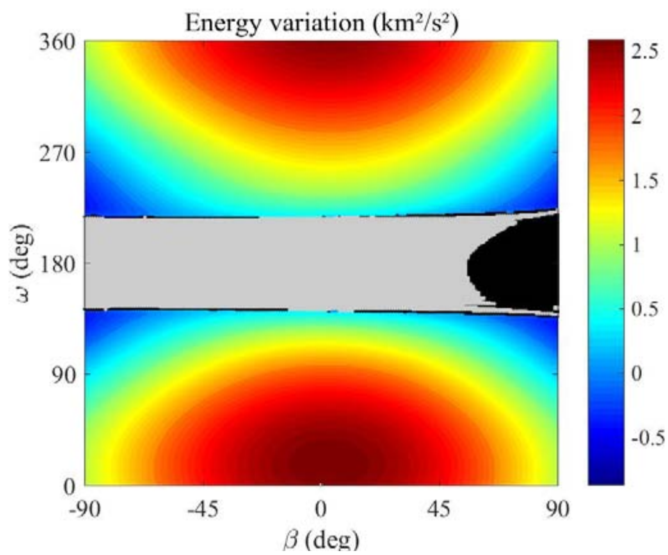
Figures 8 and 9 show equivalent results for different values for the direction of the impulse, $\eta = -45°$ and $\eta = -45°$. The results are similar, with the same interpretations.



d) $\delta V = 0.1$ km/s

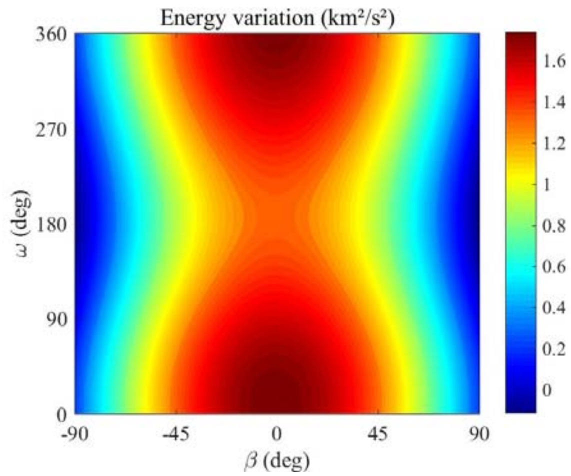


e) $\delta V = 0.3$ km/s

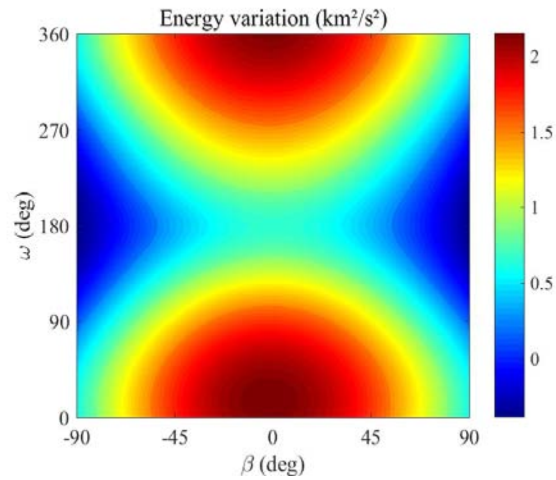


f) $\delta V = 0.5$ km/s

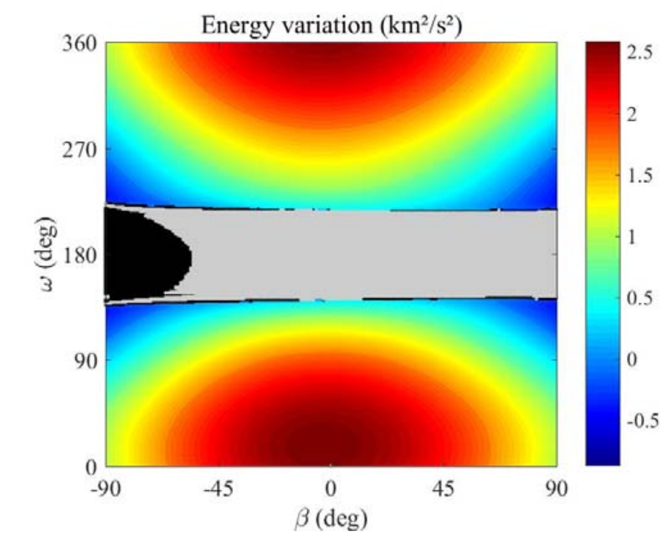
Figure 8. Energy variation for $\alpha = 270°$ and $\eta = -45°$.



a) $\delta V = 0.1$ km/s



b) $\delta V = 0.3$ km/s



c) $\delta V = 0.5$ km/s

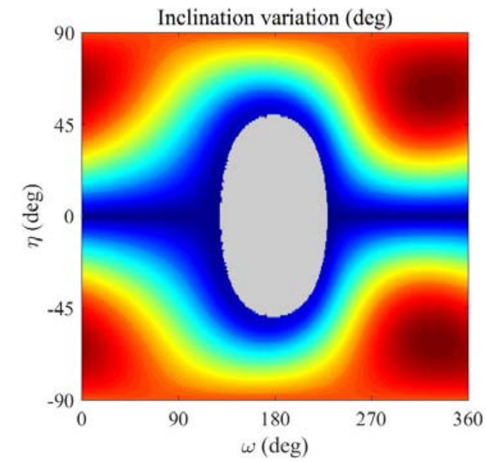
Figure 9. Energy variation for $\alpha = 270°$ and $\eta = 45°$.

VARIATIONS IN THE INCLINATION

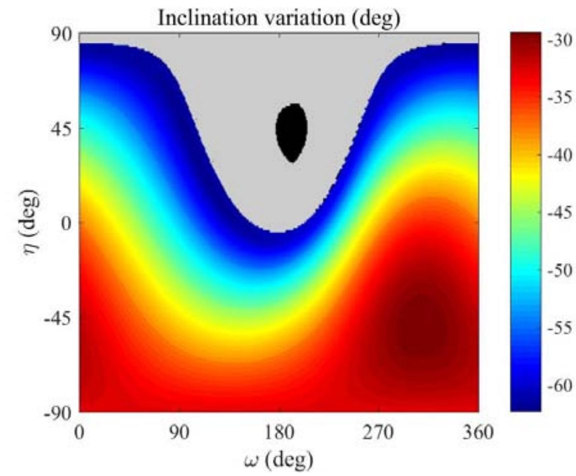
The variation of the inclination of the orbit of the spacecraft due to the powered Swing-By maneuver is an important point to be observed. To change the inclination of a spacecraft is a very expensive maneuver when made based in fuel consumption, so the use of a Swing-By to do this task is very important. Therefore, the powered Swing-By can also be used with this goal. This variation is defined as the difference between the inclination before and after the close approach. This analysis completes the study made for the energy variation, so it is possible to design a maneuver that changes energy and inclination at the same time.

Figure 10 shows the first results, when $\alpha = 270°$ (maximum gain of energy) and $\beta = 0°$ (planar approach). The first plot (Figure 10(a)) is made for the situation where the velocity of the periapsis is in the plane. If $\eta = 0°$, the whole maneuver is reduced to the plane of the primaries and the inclination of the spacecraft is zero before and after the maneuver. The range of variations in the inclination is not large, in particular compared to the situations where the spacecraft arrives with an out-of-plane component of the velocity at periapsis (Figures 10(b) and 10(c)). The reason is that, in this geometry, the variation in the inclination is made only by the out-of-plane impulse. The maximum variations reach about 5.2°, and it occur for $-90° < \eta < -45°$ and $45° < \eta < 90°$ and for impulses near the direction of the motion of the spacecraft. There is a symmetry with respect to the line $\eta = 0°$. There is also a region of collisions in the center of the plot.

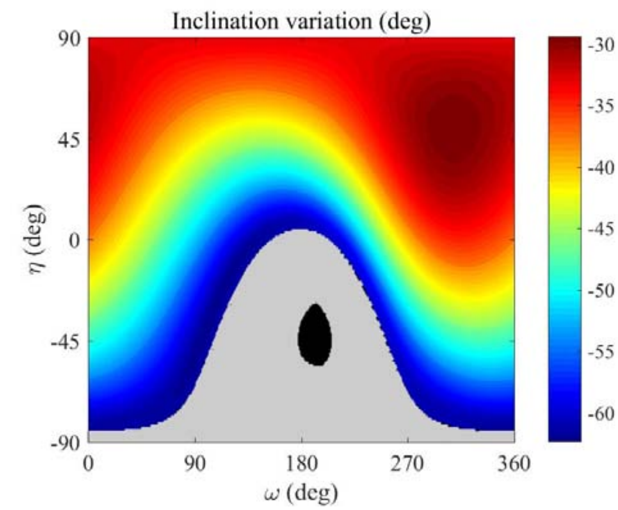
In the situations where the velocity at periapsis has an out-of-plane component, like for $\gamma = -45°$ and $\gamma = 45°$, the spacecraft always reduces its inclination after the maneuver. In these cases, shown in Figures 10(b) and 10(c), the inclination varies from $-62°$ to $-28.9°$. For the lowest reduction of inclination ($-28.9°$), ω is between 270° and 360°, so the whole maneuver takes place in the plane of the primaries, the impulse has a component in the direction of the motion of the spacecraft and another one opposite to the secondary body. The regions of captures and escapes are shifted in these situations. In Figure10(b) the impulse has another component in $V_z < 0$, and in Figure10(c) the component is in $V_z > 0$. The largest variations in inclination (dark blue region) occur for the conditions that are in the limit between captures and collisions.



a) $\gamma = 0°$



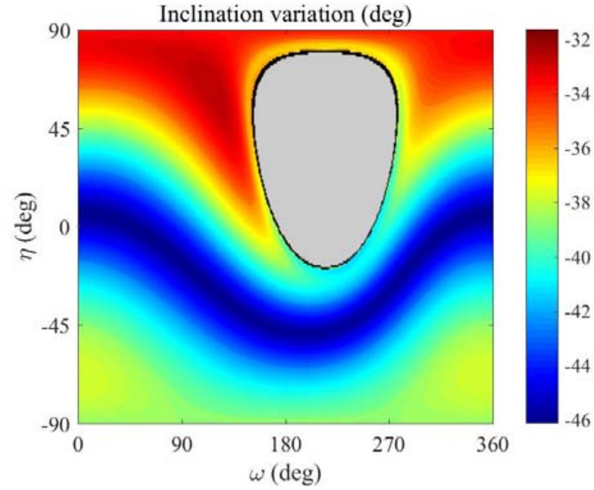
b) $\gamma = -45°$



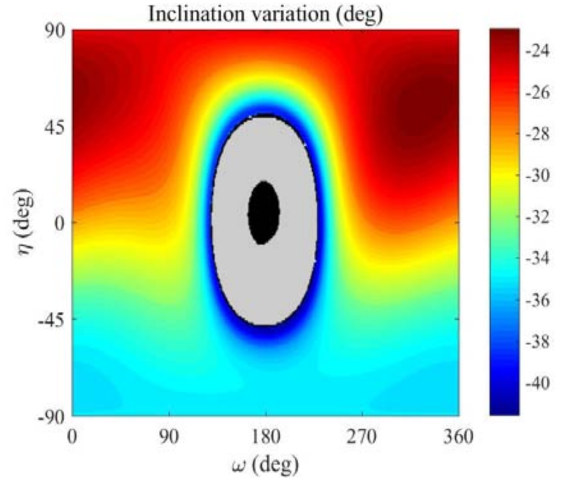
c) $\gamma = 45°$

Figure 10. Inclination variation for $\boldsymbol{\alpha} = \mathbf{270}°$ and $\boldsymbol{\beta} = \mathbf{0}°$.

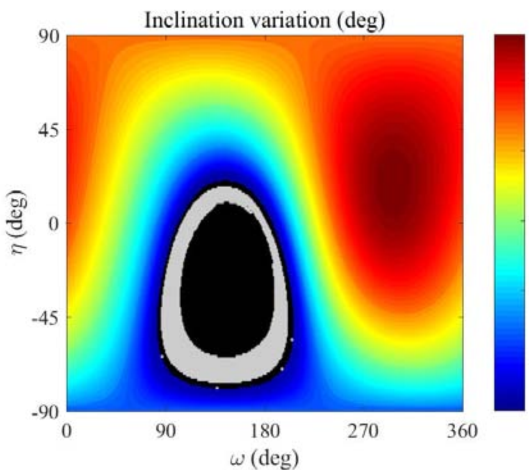
Figures 11 and 12 show the inclination variations for $\beta = -45°$ and $\beta = 45°$, respectively.



a) $\gamma = -45°$



b) $\gamma = 0°$



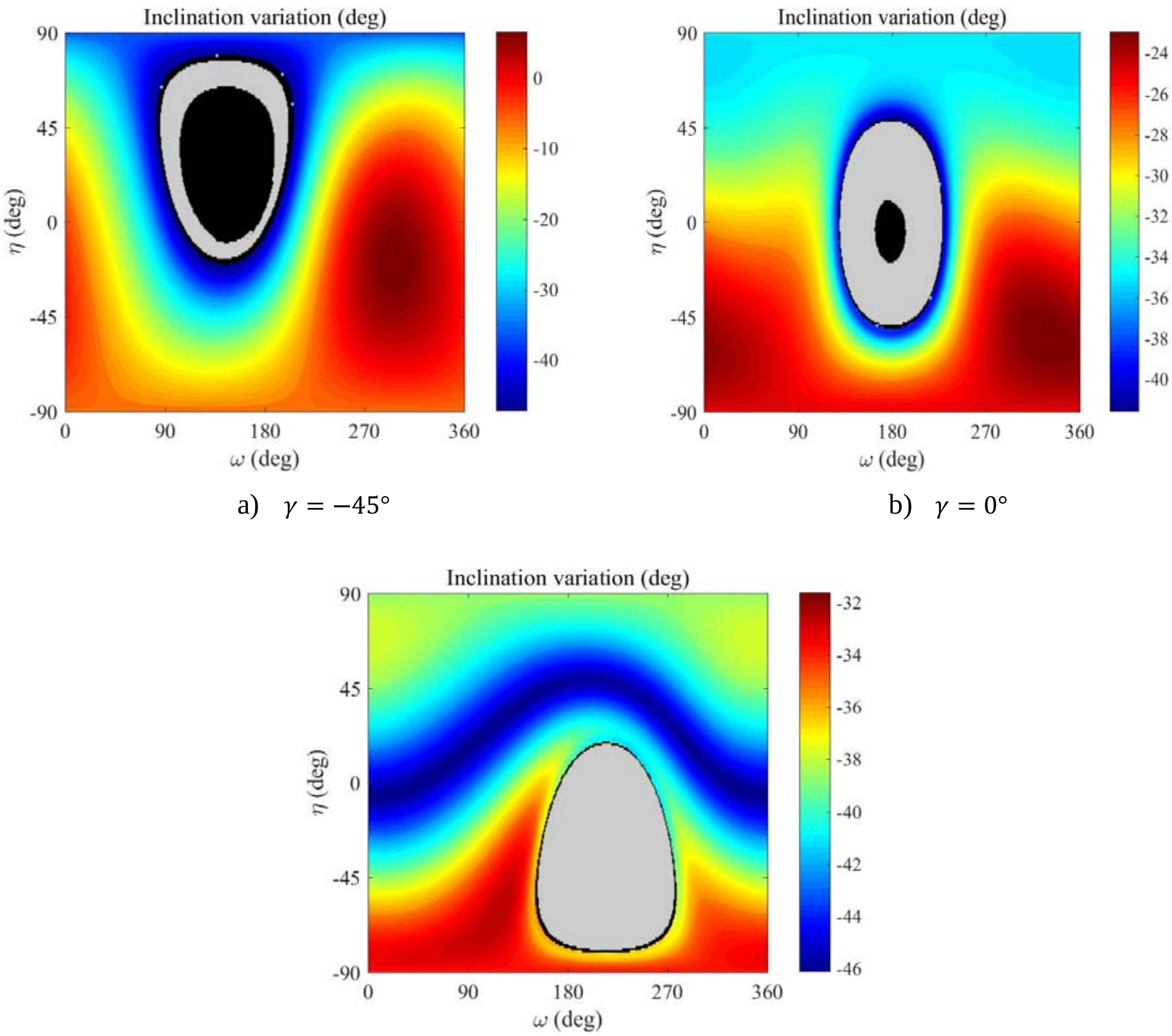
c) $\gamma = 45°$

Figure 11. Inclination variation for $\boldsymbol{\alpha = 270°}$ and $\boldsymbol{\beta = -45°}$.

Figures 11(a) and 12(c) show larger regions of collisions, always limited by regions of capture. They occur for retrograde impulses and applied in the direction opposite to M_2. For Figure 11(a), the largest variations of the inclination start at ω and η near zero, where the impulse is in the direction of the motion of the spacecraft, covers an inclined region up to ω near 180° and $\eta = -45°$, where the impulse is retrograde. Then it follows a symmetric region for $\omega > 180°$. The largest variations are of the order of 46°. For Figure 12(a) the behavior is similar, but the largest variations in magnitude are around $\omega = 180°$ and η close to 45°. The difference of the geometry of the maneuvers shown in these two figures is the velocity of the spacecraft at periapsis and the location of the periapsis, which is $\beta = -45°$ and $\gamma = -45°$ in one case and $\beta = 45°$ and $\gamma = 45°$ in the other case.

It is also observed that, when $\gamma = 0°$, Δi ranges from $-42°$ to $-22.6°$. In Figures 11(b) and 12(b), the minimum variations are in the boundary regions between captures and collisions, which occur for ω around 180° and $-45° < \eta < 45°$. The captures occur for η close to zero.

When the position and velocity of the spacecraft at the periapsis have out-of-plane components (β and γ are not zero) and pointing in opposite quadrants (Figure 11(c) and 12(a)), there are a few cases where the orbits after the maneuver is more inclined than the first one, reaching a variation of 7.5°. The largest variations are about 42° and it happens due to the inclination of the orbit before the maneuver. For this case, there is a large amount of conditions ending in captures, located in the region of retrograde impulses and pointing to M_2.



a) $\gamma = -45°$

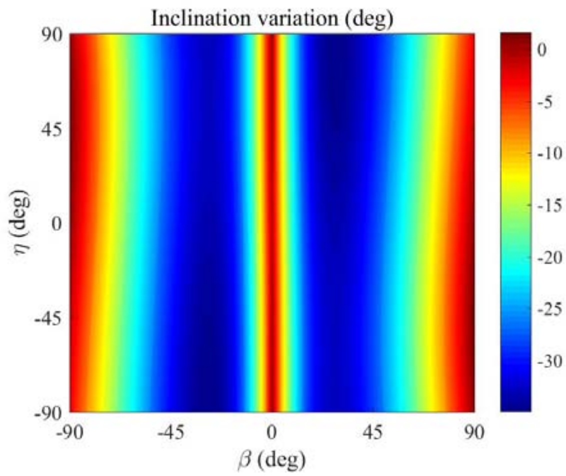
b) $\gamma = 0°$

c) $\gamma = 45°$

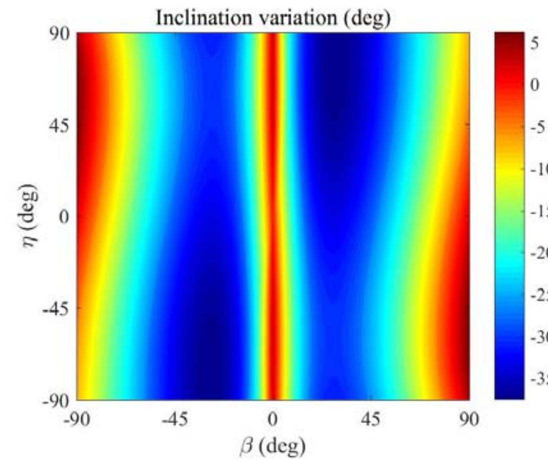
Figure 12. Inclination variation for $\alpha = 270°$ and $\beta = 45°$.

The inclusion of the impulse makes it a very complex maneuver. The impulse can dominate the maneuver for large values of their magnitudes, avoiding an expected behavior based in the analysis of the gravitational part of the maneuver. For example, the minimum energy variations occurred in the conditions studied have the highest inclinations for the first orbit, except for Figures 5(a), 8(a), 6(c) and 12(c). This same statement is not valid for cases of maximum energy variations.

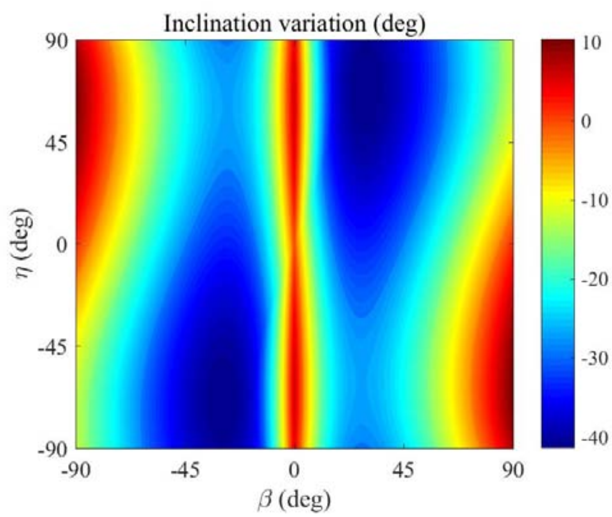
Next, we made a study considering the effects of the variables β and η. Figure 13 shows the results, which are the inclination variations for $\alpha = 270°$ (maximum gains of energy) and $\omega = 0°$ (impulse in the direction of the motion of the spacecraft). It is noted that, in Fig 13(a), made for an impulse of magnitude of 0.1 km/s, the border lines between adjacent colors are nearly vertical, which indicates that the impulse has a small participation in the variation of the inclination. Increasing the magnitude of the impulse to 0.3 km/s (Fig. 13b) and to 0.5 km/s (Fig. 13c), the impulse starts to take an important participation in the maneuvers, with the lines being no longer verticals. It is observed that the variations of inclination are zero at $\beta = 0°$ and ±90°, with maximum in the intermediate positions. The maximum variations are also located in the regions where both variables have the same sign.



a) $\delta V = 0.1$ km/s



b) $\delta V = 0.3$ km/s



c) $\delta V = 0.5$ km/s

Figure 13. Inclination variation for $\alpha = 270°$ and $\omega = 0°$

CONCLUSIONS

This research studied the powered Swing-By maneuver in the three-dimensional space. The position of the periapsis with respect to the secondary body, defined by the angle α, had a significant influence on the behavior of the variations in the energy and inclination of the spacecraft, as expected. The solutions are presented for $\alpha = 270°$, the region of maximum energy gains due to the gravitational part of the maneuver. There is symmetry in the system and the values for energy and inclination variations for $\alpha = 90°$ have the same magnitude of the ones obtained for $\alpha = 270°$, just with a reversal in the sign relative to the out-of-plane direction of the impulse (η).

Color maps showing the variations in the energy and inclination with respect to the primary body were analyzed as a function of the angles that define the impulse applied in the plane (ω) and out-of-the plane (η). Initial conditions resulting in captures and collisions with respect to M_2 occur for all cases studied. The main influence of those occurrences come from the impulse, always when it has a component opposite to the motion of the spacecraft, decelerating it to remove energy.

The energy and inclination variations are dependent of γ, which defines the out-of-plane component of the periapsis velocity. This variable has important influence on the results and is also dependent on the initial conditions adopted. In addition, considering only the gravitational part of the maneuver, the larger the γ, the smaller the effect of the Swing-By. When we consider the impulse, depending on the combination with β, there is symmetry in the solutions for ΔE and Δi with respect to the direction of the impulse (η).

The variation of the inclination, for most of the cases studied, is negative, that is, the orbit before the maneuver is more inclined than the orbit after the maneuver. An exception occurs for $\beta = 0°$ and $\gamma = 0°$, where the inclination depends only on the three-dimensional impulse and the second orbit is more inclined.

The relation between energy and inclination, when we include the impulse, does not follow exactly the same behavior for the case when only gravity is acting in the system. The minimum energy variation, for the conditions studied, occurs for the highest inclinations of the first orbit, except for some cases. The inclusion of the impulse in the maneuver makes it very complex from a dynamical point of view. The impulse can dominate the maneuver, avoiding an expected behavior made based in the analysis of only the gravitational part of the maneuver.

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