

## Orbital stability near the (87) Sylvia system

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### Abstract

The main goal of our work is to study the equatorial orbital dynamics of a spacecraft near the (87) Sylvia system. Here, we consider a non-homogeneous mass distribution with a dense core inside the primary asteroid. The Mascon gravity framework using the shaped polyhedral source, from lightcurve data, is chosen to calculate the gravitational field. The zero-velocity curves show four unstable equilibrium points. In the absence of any solar or other celestial body perturbations, a numerical analysis of the orbital dynamics in the potential field of Sylvia is done to delineate the region of stable and unstable motions.

### 1. Introduction

The 8<sup>th</sup>-largest body in the asteroid belt, (87) Sylvia, was the first asteroid known to have more than one moon. It is a triple asteroid, surrounded by two satellites, named Romulus and Remus, orbiting in nearly circular orbits in the same plane and direction, almost aligned with Sylvia’s equatorial plane. The primary body is a fairly fast rotator completing one rotation in about 5.184 h. The mass of the moons present only  $\sim 0.005\%$  of the mass of the primary body. Sylvia is the parent body of a family in the Cybele region, This family may be one of the oldest families in the extended main belt, with an age estimation of  $1220 \pm 40$  Myr old. In this way, preliminary plans to launch a spacecraft up to this triple asteroid, for spectroscopy and photometric studies to determine the internal structure, will contribute to our understanding of the early history of the Solar System.

### 2. Equations of motion

Covering a period of 100 days, the motions of the two moons of Sylvia, that lie deeply within Sylvia’s Hill sphere, and of the spacecraft are integrated with the

classical equations of motion in the body-fixed frame of reference neglecting the absence of any solar or other celestial body perturbations.

$$\begin{aligned}\ddot{x}_j - 2\omega\dot{y}_j &= \omega^2 x_j + U_{x_j} + \mathcal{A}_x(\mathcal{P}) \\ \ddot{y}_j + 2\omega\dot{x}_j &= \omega^2 y_j + U_{y_j} + \mathcal{A}_y(\mathcal{P}) \\ \ddot{z}_j &= U_{z_j} + \mathcal{A}_z(\mathcal{P}),\end{aligned}$$

where:  $i, j = 1, 2, 3$  stand for the body concerned,  $|r_i| = \sqrt{x_i^2 + y_i^2 + z_i^2}$ ,  $|r_i - r_j|$  is the distance between the bodies  $i$  and  $j$ ,  $\omega$  is the spin rate of Sylvia, and  $U_{x_j}, U_{y_j}, U_{z_j}$  are the first-order partial derivatives of the gravitation potential of the central body, calculated using the Mascon gravity approach [1, 2], dividing the shape of Sylvia into 8 equal layers. The vector  $\mathcal{P} = (\mathcal{P}_x, \mathcal{P}_y, \mathcal{P}_z)$  describes the interaction between components  $i$  and  $j$ .  $\mathcal{A}$  is an instantaneous rotation that takes the vector  $\mathcal{P}$  from an inertial frame into a body-fixed frame.

$$\begin{aligned}\mathcal{P}_x &= \sum_{i=1, i \neq j}^3 \mathcal{G}m_i \left( \frac{x_i - x_j}{|r_i - r_j|^3} - \frac{x_i}{|r_i|^3} \right) \\ \mathcal{P}_y &= \sum_{i=1, i \neq j}^3 \mathcal{G}m_i \left( \frac{y_i - y_j}{|r_i - r_j|^3} - \frac{y_i}{|r_i|^3} \right) \\ \mathcal{P}_z &= \sum_{i=1, i \neq j}^3 \mathcal{G}m_i \left( \frac{z_i - z_j}{|r_i - r_j|^3} - \frac{z_i}{|r_i|^3} \right)\end{aligned}$$

The  $J_2$  perturbations of each moon are very small compared to the central force of our system. Consequently, ignoring them will not affect the global behaviour and stability in our simulations. The initial positions of the moons were calculated from the Johnston’s Asteroids with Satellites Database, while the initial velocities are estimated using Kepler equations.

### 3. Results

12,960 massless particles orbiting Sylvia with initial periapsis distances ( $r_p$ ) between 250 and 2000 km from the primary centre with an interval of 50 km have been integrated. In the planar cases, we study particles with initially circular ( $e_{ini} = 0$ ) or slightly eccentric

prograde orbits with initial eccentricity of 0.05, 0.1, and 0.2. 90 different longitudes ( $\lambda$ ) varying from  $0^\circ$  to  $360^\circ$  are tested. For the sake of simplicity, initial conditions are chosen in such a way that each test particle is at the periapsis distance. An orbit is considered stable in this study if the variation of its periapsis radius does not exceed a threshold value (i.e. 6 km), and the variation of its eccentricity does not exceed 0.05, although the orientation of these orbits may change. Fig. 1 present an example of stable/unstable orbits close to the Sylvania. The orbit on the right-hand side is very close to the inner moon, so its mean motion is very close to that of Remus. The orbit is unstable, but it did not intersect this moon. Thus, this type of orbit could be suitable to observe the inner moon’s orbit during the close approach.

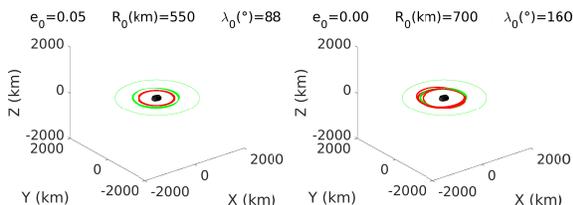


Figure 1: Example of equatorial orbits in the Sylvania-fixed frame over 100 days.

Our overall results are presented in Fig. 2. We notice that the first stable orbit is detected at a distance of 550 km from the centre of Sylvania. No collision occurs with the central body beyond 350 km. The collisions with Remus occur between 300 and 900 km, while with Romulus they occur between 900 and 1450 km. Moreover, the orbits escape from the system when the distance is smaller than 350 km. Finally, we found that the stability region around our system decreases when the initial eccentricity increases.

## 4. Summary and Conclusions

We discussed the stability of equatorial orbits close to the triple asteroidal system of (87) Sylvania. We tested the influence of three different internal structure models of the central body on orbits around the system. We found that these structures did not affect the global behavior of the stability. We presented a complete analysis considering a two-layered structure. Observing the oscillation of the periapsis and eccentricity of each orbit, we found that our stability criterion on the periapsis radius is independent of the eccentricity, i.e. the orbit with periapsis radius variation smaller than

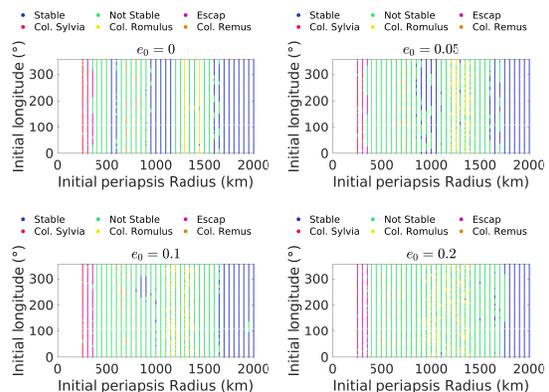


Figure 2: Stability maps relative to (87) Sylvania.

6 km is stable. However, it is not the case for the eccentricity where we found only few stable orbits with eccentricity variation smaller than 0.05. A large majority of orbits tested in this work suffer strong perturbations due to the irregular shape of Sylvania. Objects starting with perfectly circular orbits experience changes in eccentricity of 0.03. These changes are not large enough to affect the stability of an eventual probe over the mission period, but could potentially be hazardous for longer time-scales. Many stable orbits are found beyond an initial periapsis radius of 1600 km from the centre of the central. We do not identify any stable orbit with  $e_0 = 0.2$  close to Sylvania. However, some isolated regions of stable equatorial orbits with other initial eccentricities are found between the two moons (850-1150 km). Some stable low-orbits found with  $e_0 = 0.0$  occur at an initial periapsis radius of 550 km from the centre of Sylvania. Future applications of our study would be to investigate special type of orbits around (87) Sylvania, such as orbits at critical inclinations, the sun-synchronous orbits, frozen orbits, and other specific types of orbits suitable for the exploration of the triple system.

## References

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