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Unitary speed particles in circular symmetric patterns

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1 Abstract

Several living beings such as birds, fish, flies and bees perform collective motion when foraging, migrating or escaping from predators [4]. They are able to flock to a certain direction, rotate around a common center, or swarm within an area. Biologists try to understand the local rules that lead single individuals to macroscopic group behaviors [1–3].

Thereafter, such rules inspire the development of models for mobile vehicles, whose applications include data collection, surveillance and monitoring [5, 6]. Motivated by such problems, we perform extensive tests in a model of particles with phase-coupled oscillators dynamics [7], focusing on the special case in which they rotate around a common center and are able to group into clusters.

We consider particles moving in the plane with unitary mass and speed. They are coupled to each other through an all-to-all approach, meaning that every particle can see all the others. They share their positions and heading angles (phases). As result, the particles develop a concentric circular trajectory and group into uniformly distributed clusters of the same size. More details can be found in Sepulchre *et al.* [7].

We analyze the control parameters' space to find stable regions in which the clusters are well formed in a fixed integration time. The obtained formations are assessed with the so-called m-th phase order parameters [7] that tell us whether the clusters are symmetric or not. We found that the higher the number of desired clusters M, the smaller is the stable region.

Additionally, we simulate particles being added/removed to/from already stable formations. It resembles the problem of real world mobile agents that break during an ongoing

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mission and the opposite situation of new agents being included. We found transitions on the m-th phase order parameters that depend on the number of new (N_{add}) or removed (N_{rem}) particles. When $N_{new} < \delta_{add}$, or $N_{rem} > \delta_{rem}$, some particles do not join clusters and sit between stable clusters. These δ_{add} and δ_{rem} are the thresholds that point out the transition from the latter case to all particles always joining clusters.

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