

Gravitational waves from SGRs and AXPs as fast-spinning white dwarfs

Manoel F. Sousa,¹★ Jaziel G. Coelho^{1,2}★ and José C. N. de Araujo¹★

¹*Divisão de Astrofísica, Instituto Nacional de Pesquisas Espaciais, Avenida dos Astronautas 1758, 12227-010 São José dos Campos, SP, Brazil*

²*Departamento de Física, Universidade Tecnológica Federal do Paraná, 85884-000 Medianeira, PR, Brazil*

Accepted 2020 August 28. Received 2020 August 17; in original form 2020 April 15

ABSTRACT

In our previous article we have explored the continuous gravitational waves (GWs) emitted from rotating magnetized white dwarfs (WDs) and their detectability by the planned GW detectors such as Laser Interferometer Space Antenna (LISA), Deci-hertz Interferometer Gravitational wave Observatory (DECIGO), and Big Bang Observer (BBO). Here, GWs' emission due to magnetic deformation mechanism is applied for soft gamma repeaters (SGRs) and anomalous X-ray pulsars (AXPs), described as fast-spinning and magnetized WDs. Such emission is caused by the asymmetry around the rotation axis of the star generated by its own intense magnetic field. Thus, for the first time in the literature, the GW counterparts for SGRs/AXPs are described as WD pulsars. We find that some SGRs/AXPs can be observed by the space detectors BBO and DECIGO. In particular, 1E 1547.0–5408 and SGR 1806–20 could be detected in 1 yr of observation, whereas SGR 1900+14, CXOU J171405.7–381031, Swift J1834.9–0846, SGR 1627–41, PSR J1622–4950, SGR J1745–2900, and SGR 1935+2154 could be observed with a 5-yr observation time. The sources XTE J1810–197, SGR 0501+4516, and 1E 1048.1–5937 could also be seen by BBO and DECIGO if these objects have $M_{\text{WD}} \lesssim 1.3 M_{\odot}$ and $M_{\text{WD}} \lesssim 1.2 M_{\odot}$, respectively. We also found that SGRs/AXPs as highly magnetized neutron stars are far below the sensitivity curves of BBO and DECIGO. This result indicates that a possible detection of continuous GWs originated from these objects would corroborate the WD pulsar model.

Key words: gravitational waves – stars: magnetic field – white dwarfs.

1 INTRODUCTION

Over the last decade, there has been an increasing interest of the astrophysics community on highly magnetized white dwarfs (HMWDs) both from the theoretical and observational points of view. These sources constitute at least 10 per cent of the white dwarfs (WDs) if observational biases are considered (Kawka et al. 2007). These WDs with surface magnetic fields ranging from 10^6 to 10^9 G have been confirmed by the recent results of the Sloan Digital Sky Survey (Külebi et al. 2009; Kepler et al. 2010, 2013, 2015). Besides their high magnetic fields, most of them have been shown to be massive and responsible for the high-mass peak at $1 M_{\odot}$ of the WD mass distribution; for instance: REJ 0317–853 has $M \approx 1.35 M_{\odot}$ and $B \approx (1.7\text{--}6.6) \times 10^8$ G (Barstow et al. 1995; Külebi et al. 2010); PG 1658+441 has $M \approx 1.31 M_{\odot}$ and $B \approx 2.3 \times 10^6$ G (Liebert et al. 1983; Schmidt et al. 1992); and PG 1031+234 has the highest magnetic field $B \approx 10^9$ G (Schmidt et al. 1986; Külebi et al. 2009). The existence of ultramassive WDs has been revealed in several studies (Althaus et al. 2005, 2007; Castanheira et al. 2013; Hermes et al. 2013; Curd et al. 2017; Camisassa et al. 2019; Gentile Fusillo et al. 2018; Jiménez-Esteban et al. 2018).

Typically, WDs rotate with periods of days or even years. Recently, a WD pulsar known as AR Scorpii was discovered with a period of 1.97 min, emitting radiation in a broad range of frequencies, typically

of neutron star (NS) pulsars (Marsh et al. 2016). The spin-down power is an order of magnitude larger than the observed luminosity (dominated by the X-rays), which, together with an absence of obvious signs of accretion, suggests that AR Sco is primarily rotation powered. The AR Sco's broad-band spectrum is characteristic of synchrotron radiation, requiring relativistic electrons, possibly originated from the neighbourhood of the WD and accelerated to almost the speed of light (Lobato, Coelho & Malheiro 2017). Furthermore, other sources have been proposed as candidates of WD pulsars. A specific example is AE Aquarii, the first WD pulsar identified, with a short rotation period of $P = 33.08$ s (Terada et al. 2008b) and spinning down at a rate $\dot{P} = 5.64 \times 10^{-14}$ s s⁻¹. The rapid braking of the WD and the nature of hard X-ray pulses detected with *Suzaku* space telescope (Terada et al. 2008a) can be explained in terms of spin-powered pulsar mechanism (see Ikhsanov 1998). On the other hand, the *X-ray Multimirror Mission (XMM)*–*Newton* satellite has observed a WD faster than AE Aquarii. Mereghetti et al. (2009) showed that the X-ray pulsator RX J0648.0–4418 is a massive WD with mass $M = 1.28 M_{\odot}$ and radius $R = 3000$ km (see Althaus et al. 2005, 2007 for derived mass–radius relations for massive oxygen–neon WDs that predict this radius), with a very fast spin period of $P = 13.2$ s, that belongs to the binary system HD 49798/RX J0648.0–4418. More recently, Lopes de Oliveira et al. (2020) report on *XMM*–*Newton* observations that reveal CTCV J2056–3014 to be an X-ray-faint intermediate polar harbouring an extremely fast-spinning WD with a coherent 29.6 s pulsation.

* E-mail: manoelfelipesousa@gmail.comj (MFS); azielcoelho@utfpr.edu.br (JGC); jcarlos.dearaujo@inpe.br (JCNdA)

Table 1. Observational quantities taken from McGill Pulsar Group’s online catalogue for confirmed SGRs/AXPs: Period (P), spin-down (\dot{P}), observed luminosity (L_X), and distance to the source (r).

SGR/AXP	P (s)	\dot{P} (10^{-11} s s $^{-1}$)	L_X (10^{33} erg s $^{-1}$)	r (kpc)
CXOU J010043.1–721134	8.020392	1.88	65	62.4
4U 0142+61	8.688692	0.2022	105	3.6
SGR 0418+5729	9.078388	0.0004	0.00096	2
SGR 0501+4516	5.76207	0.594	0.81	2
SGR 0526–66	8.0544	3.8	189	53.6
1E 1048.1–5937	6.457875	2.25	49	9
1E 1547.0–5408	2.072126	4.77	1.3	4.5
PSR J1622–4950	4.3261	1.7	0.44	9
SGR 1627–41	2.594578	1.9	3.6	11
CXOU J164710.2–455216	10.61064	≤ 0.04	0.45	3.9
1RXS J170849.0–400910	11.00502	1.9455	42	3.8
CXOU J171405.7–381031	3.825352	6.40	56	13.2
SGR J1745–2900	3.763638	1.385	≤ 0.11	8.3
SGR 1806–20	7.54773	49.5	163	8.7
XTE J1810–197	5.540354	0.777	0.043	3.5
Swift J1822.3–1606	8.437721	0.0021	≤ 0.00040	1.6
SGR 1833–0832	7.565408	0.35	...	$\leq 10^a$
Swift J1834.9–0846	2.482302	0.796	≤ 0.0084	4.2
1E 1841–045	11.78898	4.092	184	8.5
J185246.6+003317	11.55871	≤ 0.014	≤ 0.0060	7.1
SGR 1900+14	5.19987	9.2	90	12.5
SGR 1935+2154	3.245065	1.43	...	$\leq 10^b$
1E 2259+586	6.979043	0.0483	17	3.2

Notes. ^aSee Esposito et al. (2011); ^bsee Kozlova et al. (2016).

Notwithstanding, several current studies of fast-rotating and magnetized WDs have been done, in particular the one involving WD pulsars in an alternative description for soft gamma repeaters (SGRs) and anomalous X-ray pulsars (AXPs; see Malheiro, Rueda & Ruffini 2012; Coelho & Malheiro 2014; Lobato, Malheiro & Coelho 2016; Mukhopadhyay & Rao 2016; Cáceres et al. 2017, and references therein). From this perspective, a canonical spin-powered pulsar model can explain the process of energy emission released by dipole radiation in a WD, since they share quite similar aspects (Usov 1988; Coelho & Malheiro 2014). In addition, these sources could also be candidates for GW emission, since the huge magnetic field can deform the star in a non-symmetrical way, thus generating a variation in the quadrupolar moment of the star.

In a second proposed scenario, that of a WD pulsar, the optical/IR data are explained by the WD photosphere and by a disc (Rueda et al. 2013). Recently, a new scenario has been proposed to explain the spectral energy distribution of 4U 0142+61, from mid-infrared up to hard X-rays (Borges et al. 2020). In this model, the persistent emission comes from an accreting isolated magnetic WD surrounded by a debris disc, having gas and dusty regions.

On the other hand, direct observations of GWs have recently been made by Laser Interferometer Gravitational-Wave Observatory (LIGO) and Virgo. The first event was detected in 2015 by LIGO (Abbott et al. 2016). This event, named GW150914, came from the merging of two black holes of masses ~ 35.6 and $30.6 M_\odot$ that resulted in a black hole of mass $\sim 63.1 M_\odot$. Thereafter, LIGO in collaboration with Virgo observed nine more such events (Abbott et al. 2017a, b, d, 2019). In addition, the event GW170817 reports the first detection of GW from a binary NS inspiral (Abbott et al. 2017c). All GW detections are within a frequency band ranging from 10 to 1000 Hz, which is the operating band of LIGO and Virgo. As is well known, there are proposed missions for lower

frequencies, such as LISA (Amaro-Seoane et al. 2017; Robson, Cornish & Liug 2019), whose frequency band is of (10^{-4} – 0.01) Hz, BBO (Harry et al. 2006; Yagi & Seto 2011) and DECIGO (Kawamura et al. 2006; Yagi & Seto 2017) in the frequency band ranging from 0.01 to 10 Hz.

Different possibilities of generation of continuous GWs have already been proposed (see e.g. Bonazzola & Gourgoulhon 1996; De Araujo, Coelho & Costa 2016a, b, c, 2017; Gao, Cao & Zhang 2017; Franzone & Schramm 2017; Mukhopadhyay, Rao & Bhatia 2017; Pereira, Coelho & de Lima 2018; De Araujo et al. 2019, and references therein). More recently, Kalita & Mukhopadhyay (2019) show that continuous GWs can be emitted from rotating magnetized WDs and will possibly be detected by the upcoming GW detectors such as LISA, DECIGO, and BBO. The main goal of this paper is to extend our previous study (Sousa, Coelho & de Araujo 2020) in which we investigated the gravitational radiation from three fast-spinning magnetized WDs, which have high rotations (a few seconds to minutes) and huge magnetic fields (10^8 – 10^9 G), considering two emission mechanisms: matter accretion and magnetic deformation. In both cases, the GW emission is generated by asymmetry around the rotation axis of the star due to accumulated mass in the magnetic poles and due to the intense magnetic field, respectively. Here, we explore the magnetic deformation mechanism of gravitational radiation emission in SGRs/AXPs as fast-spinning magnetized WD.

This paper is organized as follows. In Section 2, we present the aspects of the model used to explain SGRs/AXPs. In Section 3, we describe the mechanism of GW emission by deriving the equations for the gravitational amplitude and luminosity. In Section 4, we present and discuss the calculations applied to SGRs/AXPs described as WD pulsars. Finally, in Section 5, we summarize the main conclusions and remarks.

2 SGRS/AXPS AS WHITE DWARF PULSARS

SGRs and AXPs are a special class of pulsars that present distinct characteristics from radio pulsars and X-ray pulsars (see Table 1). They are described by the magnetar model, where they are considered strongly magnetized NS with magnetic field of the order of 10^{12} – 10^{15} G. Also, these objects are known as very slow rotating pulsars comparing to ordinary pulsars, with rotational periods in the range of $P \sim 2$ – 12 s and a high spin-down rate of $\dot{P} \sim 10^{-13}$ – 10^{-10} s s $^{-1}$ (see Olausen & Kaspi 2014, and references therein).¹

Recently, three SGRs with low magnetic field ($B \sim 10^{12}$ – 10^{13} G) have been observed, namely, SGR 0418+5729, Swift J1822.3–1606, and 3XMM J185246.6+00331. These new discoveries open the question concerning the nature of SGRs/AXPs, emerging alternative scenarios, in particular the WD pulsar model. These astronomical observations have based an alternative description of the SGRs/AXPs, which are modelled as rotating highly magnetized and very massive WDs (see Malheiro et al. 2012; Coelho & Malheiro 2014; Cáceres et al. 2017, for further details). From this perspective, a canonical spin-powered pulsar model can explain the process of energy emission released by a dipole radiation in a WD, since they share quite similar aspects (see Usov 1988). In this new description, several observational properties are explained as a consequence of the large radius of a massive WD that manifests a new scale of mass density, moment of inertia, rotational energy, and magnetic dipole moment in comparison with the case of NSs (see e.g. Coelho & Malheiro 2014; Lobato et al. 2016, and references therein).

In the canonical pulsar model, a rotating star with magnetic dipole moment misaligned to the axis of rotation converts rotational energy into electromagnetic energy. Thus, the system emits radiation due to the variation of the magnetic dipole and the pulsar rotation becomes slower. If we consider that all rotational energy loss is converted to electromagnetic energy, we can infer the magnetic field strength on the star's surface, B_s , as a function of the period $P = 1/f_{\text{rot}}$ and its derivative $\dot{P} = dP/dt$ (see e.g. Coelho & Malheiro 2014):

$$B_s \sin \phi = \left(\frac{3c^3 I}{8\pi^2 R^6} P \dot{P} \right)^{1/2}, \quad (1)$$

where I is the moment of inertia, R is the radius of the star, ϕ is the unknown angle between the rotation and magnetic dipole axes, and c is the speed of light.

Note that because the moment of inertia values for a NS and a WD are different, the magnetic fields required in each model are also different. For example, for a NS with mass $M = 1.4 M_\odot$ and radius $R = 10^6$ cm, the magnetic field on the star's surface are in the range of 10^{13} – 10^{15} G. For a very massive WD with mass $M = 1.4 M_\odot$ and radius $R \sim 10^8$ cm (see e.g. Coelho & Malheiro 2014), the magnetic field has smaller values and is in a range around 10^9 – 10^{11} G, comparable to the inferred values of known HMWDs (Külebi et al. 2009; Kepler et al. 2010, 2013, 2015). Thus, these values of mass and radius generate a moment of inertia $I \simeq 1.1 \times 10^{49}$ g cm 2 . These results clearly show that the scale of the magnetic field in WD is 10^4 times smaller than for NSs.

In addition, since the high magnetic field can deform the star in a non-symmetrical way, new values for the ellipticity are expected and, consequently, new values for the GW amplitude as well (see Section 3). In the next section, we describe the magnetic deformation

mechanism and deduce the GW amplitude and luminosity emitted in this process.

3 MAGNETIC DEFORMATION MECHANISM: BASIC EQUATIONS

WDs might generate GWs whether they are not perfectly symmetric around their rotation axes. This asymmetry can occur, for example, due to the huge dipole magnetic field that can make the star become oblate (see e.g. Chandrasekhar & Fermi 1953). In this work, we analyse the emission of gravitational radiation from SGRs/AXPs as fast-spinning magnetized WDs by this mechanism.

Thus, in this section we consider the deformation of the WDs induced by their own huge magnetic fields. Due to the combination of magnetic field and rotation, a WD can become triaxial, presenting therefore a triaxial moment of inertia. In order to investigate the effect arising from the magnetic stress on the equilibrium configuration of the stars, let us introduce the equatorial ellipticity, defined as follows (see e.g. Shapiro & Teukolsky 1983; Maggiore 2008):

$$\epsilon = \frac{I_1 - I_2}{I_3}, \quad (2)$$

where I_1 , I_2 , and I_3 are main moments of inertia with respect to the x , y , and z axes, respectively.

If the star rotates around the z -axis, then it will emit monochromatic GWs with a frequency twice the rotation frequency, f_{rot} , and amplitude given by (see e.g. Shapiro & Teukolsky 1983; Maggiore 2008)

$$h_{\text{df}} = \frac{16\pi^2 G}{c^4} \frac{I_3 f_{\text{rot}}^2}{r} \epsilon \quad (3)$$

and the rotational energy of the star decreases at a rate given by (see e.g. Shapiro & Teukolsky 1983; Maggiore 2008)

$$L_{\text{GWdf}} = -\frac{2^{11}\pi^6}{5} \frac{G}{c^5} I_3^2 \epsilon^2 f_{\text{rot}}^6. \quad (4)$$

On the other hand, recall that the ellipticity of magnetic origin can be written as follows (see e.g. Bonazzola & Gourgoulhon 1996; Konno, Obata & Kojima 2000; Regimbau & de Freitas Pacheco 2006):

$$\epsilon = \kappa \frac{B_s^2 R^4}{GM^2} \sin^2 \phi, \quad (5)$$

where, as before, B_s is the magnetic field strength on the star's surface, R and M are, respectively, the radius and mass of the star, ϕ is the angle between the rotation and magnetic axes, whereas κ is the distortion parameter, which depends on the magnetic field configuration and equation of state (EoS) of the star. It is worth mentioning that as the magnetic field of the SGRs/AXPs is inferred from the spin-down rate of the star, where it is considered that all rotational energy loss is converted into electromagnetic energy (see equation 1), care must be taken when using these field values to calculate an additional GW spin-down torque. In fact, in the next section, we show that this procedure is safe and that only a small fraction of the total energy loss goes to GWs.

To proceed, substituting this last equation into equations (3) and (4) and considering $I_3 = 2MR^2/5$, one immediately obtains that

$$h_{\text{df}} = \frac{32\pi^2}{5c^4} \frac{R^6 f_{\text{rot}}^2}{rM} \kappa (B_s \sin \phi)^2 \quad (6)$$

and

$$L_{\text{GWdf}} = -\frac{2^{13}\pi^6}{5^3 c^5} \frac{R^{12} f_{\text{rot}}^6}{GM^2} \kappa^2 (B_s \sin \phi)^4. \quad (7)$$

¹For information about the SGRs/AXPs, we refer the reader to the McGill University's online catalogue available at: <http://www.physics.mcgill.ca/~pulsar/magnetar/main.html>.

Table 2. Parameters for SGRs/AXPs based on a very massive white dwarf of fiducial parameters, $M_{\text{WD}} = 1.4 M_{\odot}$ and radius $R_{\text{WD}} = 1.0 \times 10^8$ cm (Boshkayev et al. 2013b). We adopt in the calculation $\kappa = 10$.

SGRs/AXPs	SGRs/AXPs as massive WD of $M_{\text{WD}} = 1.4 M_{\odot}$				
	$B_s \sin \phi$ (10^{10} G)	ϵ (10^{-6})	L_{sd} (10^{37} erg s $^{-1}$)	L_{GWdf} (erg s $^{-1}$)	η_{df} (10^{-10})
CXOU J010043.1–721134	4.12	3.28	1.59	5.48×10^{27}	3.43
4U 0142+61	1.41	0.382	0.135	4.59×10^{25}	0.340
SGR 0418+5729	0.0640	0.000790	0.000234	1.51×10^{20}	0.000645
SGR 0501+4516	1.96	0.745	1.36	2.05×10^{27}	1.51
SGR 0526–66	5.87	6.66	3.19	2.20×10^{28}	6.91
1E 1048.1–5937	4.05	3.16	3.66	1.87×10^{28}	5.10
1E 1547.0–5408	3.34	2.15	235.0	7.92×10^{30}	33.7
PSR J1622–4950	2.89	1.60	9.20	5.30×10^{28}	5.75
SGR 1627–41	2.36	1.07	47.7	5.11×10^{29}	10.7
CXOU J164710.2–455216	0.692	0.0924	0.0147	8.10×10^{23}	0.0552
1RXS J170849.0–400910	4.92	4.67	0.641	1.66×10^{27}	2.59
CXOU J171405.7–381031	5.25	5.33	50.1	1.23×10^{30}	24.5
SGR J1745–2900	2.43	1.14	11.4	6.18×10^{28}	5.41
SGR 1806–20	20.5	81.3	50.4	4.84×10^{30}	96.0
XTE J1810–197	2.20	0.937	2.00	4.11×10^{27}	2.05
Swift J1822.3–1606	0.141	0.00386	0.00153	5.58×10^{21}	0.00365
SGR 1833–0832	1.73	0.576	0.354	2.40×10^{26}	0.677
Swift J1834.9–0846	1.49	0.430	22.8	1.07×10^{29}	4.70
1E 1841–045	7.37	10.5	1.09	5.56×10^{27}	5.08
3XMM J185246.6+003317	0.427	0.0352	0.00397	7.05×10^{22}	0.0177
SGR 1900+14	7.34	10.4	28.7	7.43×10^{29}	25.9
SGR 1935+2154	2.29	1.01	18.3	1.18×10^{29}	6.45
1E 2259+586	0.617	0.0735	0.0624	6.34×10^{24}	0.102

Thereby, we find equations for the gravitational luminosity and the GW amplitude which depend on the rotation frequency and the magnetic field strength.

A very interesting equation can be obtained by substituting equation (1) into equation (5), namely

$$\epsilon = \frac{3}{20\pi^2} \frac{c^3}{GM} P \dot{P} \kappa, \quad (8)$$

which is independent of the angle ϕ .

Consequently, by substituting equation (8) into equations (6) and (7) one obtains

$$h_{\text{df}} = \frac{24}{25} \frac{R^2}{c r} \frac{\dot{P}}{P} \kappa \quad (9)$$

and

$$L_{\text{GWdf}} = -\frac{2^9 3^2 \pi^2}{5^5} \frac{c}{G} R^4 \left(\frac{\dot{P}}{P^2} \right)^2 \kappa^2. \quad (10)$$

Therefore, it is not necessary to be concerned about ϕ in the calculations of the luminosity and amplitude of GWs. Different values of ϕ are only important in the calculation of B_s . Note, however, that ϕ and B_s are not independent, as can be seen from equation (1).

Another point to note here is that the GW amplitude (equation 9) depends on the square of the radius. However, it is worth recalling that for compact stars, as WDs, the mass is related to the radius so that the more massive the star is, the smaller the radius will be. Thus, the amplitude implicitly depends on the mass M .

One could argue that rotation could be also important in the deformation of the star. In fact, rotation modifies the equatorial radius of the star, as a result the amplitude of the GWs is affected, since $h_{\text{df}} \propto R^2$. Thus, the higher the rotation velocity is, the greater is the radius.

Now, we are ready to calculate the GW amplitude and luminosity for SGRs/AXPs as massive fast-spinning WDs. The next section is

devoted to this issue as well as the corresponding discussion of the results.

4 RESULTS AND DISCUSSIONS

Here we consider that SGRs/AXPs are fast-spinning and magnetized WDs which emit GWs due to the deformation caused by their own intense magnetic field. For this study, we use the magnetic field values inferred from the canonical pulsar model (see equation 1), where it is considered that all the spin-down luminosity of the star is converted to electromagnetic luminosity.

Thus, using equation (7), we calculate the GW luminosity for several SGRs/AXPs, considering these objects as a very massive WD of $M_{\text{WD}} = 1.4 M_{\odot}$ and radius $R_{\text{WD}} = 1.0 \times 10^8$ cm (Boshkayev et al. 2013b). We adopted $\kappa \simeq 10$, which is a conservative value that holds for an incompressible fluid star with a dipole magnetic field (see e.g. Ferraro 1954). Similar values are obtained when relativistic models based on a polytropic EoS with dipole magnetic field are considered (see e.g. Konno et al. 2000). In fact, this parameter can take different values depending on the EoS and the geometry of the magnetic field (see e.g. Regimbau & de Freitas Pacheco 2006 for a useful discussion regarding this issue).

The result of this calculation is presented in Table 2, which also displays the ellipticity ϵ , the spin-down luminosity $L_{\text{sd}} (= 4\pi^2 I_3 f_{\text{rot}} \dot{f}_{\text{rot}})$ and the efficiency $\eta_{\text{df}} (= L_{\text{GWdf}}/L_{\text{sd}})$. Notice that the efficiencies are around 10^{-9} – 10^{-13} . This implies that the gravitational luminosity is much smaller than the spin-down luminosity when considering the magnetic fields inferred by the dipole model. Thus, we see that we can apply these magnetic field values to calculate the GW amplitude, since the emission of gravitational energy is negligible as compared to the rotational energy loss rate, not changing significantly the inferred magnetic fields.

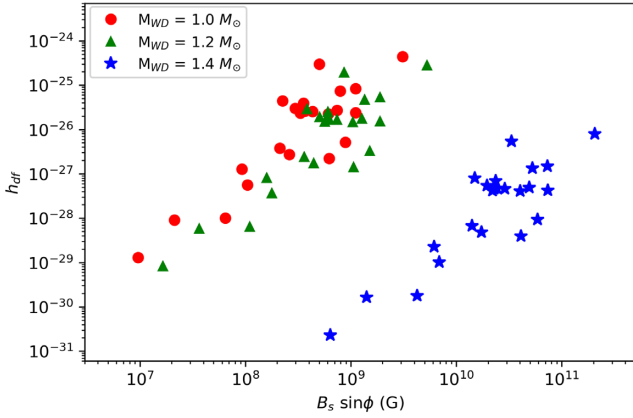


Figure 1. GW amplitude as a function of the magnetic field for SGRs/AXPs as fast-spinning and magnetized WDs.

It is worth recalling that the rotation periods as short as the ones observed in SGRs/AXPs, which ranges from $P \sim 2$ to 12 s, can be really attained by WDs. Boshkayev et al. (2013b) showed that the range of minimum rotation periods of massive WDs is of $0.3 \leq P \leq 2.2$ s, depending on the nuclear composition.² We refer the reader to the paper of these very authors for further discussion and details.

Fig. 1 shows the GW amplitudes versus the magnetic fields for the 23 confirmed SGRs/AXPs (see also Table 1). Besides our fiducial WD model ($M_{\text{WD}} = 1.4 M_{\odot}$ of $R_{\text{WD}} = 1.0 \times 10^8$ cm), we consider two additional models, namely, $1.2 M_{\odot}$ [$R_{\text{WD}} = 6.0 \times 10^8$ cm] and $1.0 M_{\odot}$ [$R_{\text{WD}} = 7.5 \times 10^8$ cm] (see Boshkayev et al. 2013a for further details about the mass–radius relation). We stress that the stability of rotating WDs was analysed taking into account the mass-shedding limit, inverse β -decay, and pycnonuclear instabilities (see also Coelho et al. 2014 for several macro and micro instabilities in WDs), as well as the secular axisymmetric instability (see fig. 1 of Boshkayev et al. 2013a for details). Note that for a given source, the predicted magnetic field (GW amplitude) can vary almost two orders of magnitude depending on the assumed parameters.

Fig. 2 shows the GW amplitude as a function of frequency for some SGRs/AXPs, where the bullets stand for $M_{\text{WD}} = 1.2 M_{\odot}$ and the vertical bars, that crosses the bullets, stand for $1.0 M_{\odot} \leq M_{\text{WD}} \leq 1.4 M_{\odot}$, from top to bottom. We also plot the sensitivity curves for BBO and DECIGO. It is worth mentioning that to plot the sensitivity curves, we use the minimum amplitude, h_{min} , that can be measured by the detector, for a periodic signal, for a given signal-to-noise ratio (SNR) and observation time T (see e.g. Maggiore 2008 for further details). Thereby, Fig. 2 presents the GW amplitudes for the sources (h_{af}) and the sensitivity curves are set to $\text{SNR} = 8$ and $T = 1$ yr. Note that we do not display the sensitivity curve for LISA because these sources are far below it.

Notice that some SGRs/AXPs produce GWs with amplitudes that can be detected by BBO and DECIGO. For example, 1E 1547.0–5408 and SGR 1806–20 could well be detected for the entire mass range considered. SGR 1900+14 and CXOU J171405.7–381031, in turn, are detectable for the entire mass range only by BBO. For these sources to be observed by DECIGO, they

²The relatively long minimum period of ^{56}Fe spinning WDs, ~ 2.2 s, implies that spinning WDs describing SGRs/AXPs have to be composed of nuclear compositions lighter than ^{56}Fe , e.g. ^{12}C or ^{16}O (see Boshkayev et al. 2013b for details).

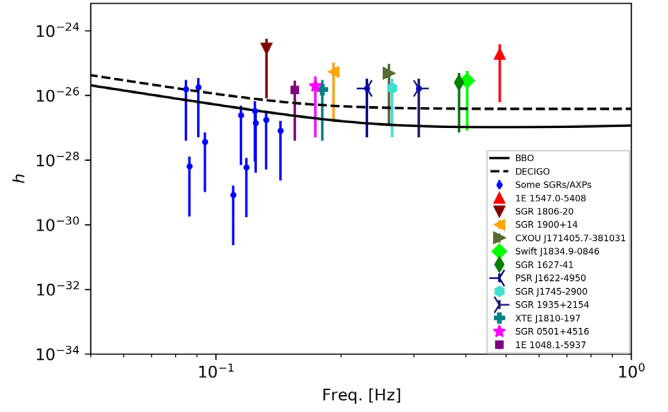


Figure 2. GW amplitude as a function of frequency for SGRs/AXPs as fast-spinning and magnetized WDs for masses in the interval $1.0 M_{\odot} \leq M_{\text{WD}} \leq 1.4 M_{\odot}$ (or $7.5 \times 10^8 \text{ cm} \geq R_{\text{WD}} \geq 1.0 \times 10^8 \text{ cm}$), represented by the vertical bars, from top to bottom. The bullets stand for $M_{\text{WD}} = 1.2 M_{\odot}$ ($R_{\text{WD}} = 6.0 \times 10^8 \text{ cm}$). Also plotted are the sensitivity curves for BBO and DECIGO for $\text{SNR} = 8$ and integration time $T = 1$ yr.

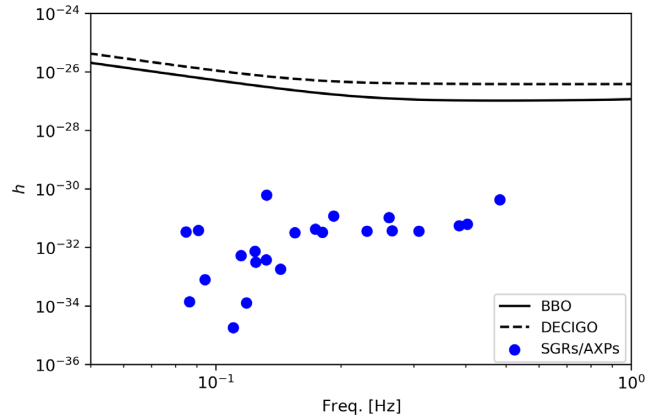


Figure 3. GW amplitude as a function of frequency for SGRs/AXPs as NSs. Also plotted are the sensitivity curves for BBO and DECIGO for $\text{SNR} = 8$ and integration time $T = 1$ yr. We consider an NS of $M = 1.4 M_{\odot}$, radius $R = 10$ km, and ellipticity given by $\epsilon = 10B^2 R^4 \sin^2 \phi / GM^2$ from De Araujo et al. (2017).

must have mass $M_{\text{WD}} \lesssim 1.3 M_{\odot}$. The sources Swift J1834.9–0846, SGR 1627–41, PSR J1622–4950, SGR J1745–2900, and SGR 1935+2154 could be observed by BBO and DECIGO if they have mass $M_{\text{WD}} \lesssim 1.3 M_{\odot}$ and $M_{\text{WD}} \lesssim 1.2 M_{\odot}$, respectively. However, if these SGRs/AXPs have $M_{\text{WD}} \sim 1.4 M_{\odot}$, they should be detectable only by BBO and with an integration time of $T = 5$ yr. XTE J1810–197, SGR 0501+4516, and 1E 1048.1–5937 could also be seen by BBO and DECIGO if these objects have $M_{\text{WD}} \lesssim 1.3 M_{\odot}$ and $M_{\text{WD}} \lesssim 1.2 M_{\odot}$, respectively, but they will not be observed if they have mass $M_{\text{WD}} \sim 1.4 M_{\odot}$, even considering $T = 5$ yr.

Therefore, SGRs/AXPs described as WDs, which have moments of inertia four orders of magnitude greater than an NS, would generate GW amplitudes much larger than SGRs/AXPs described as NSs (see Fig. 3). Consequently, if these sources are NSs, the GW amplitudes generated are far below the sensitivity curves of BBO and DECIGO. Thus, if these space-based instruments observe continuous GWs from these SGRs/AXPs, this would corroborate the model of fast-spinning and magnetic WDs. This supports the description of SGR

and AXPs as belonging to a class of very fast and magnetic massive WDs in perfect accord with recent astronomical observations of HMWDs.

It is worth noticing that from equation (9) that $h_{df} \propto \kappa$. Thus, if we consider an even more conservative value for κ , e.g. of the order of the unit, the GW amplitude decreases an order of magnitude and some sources in Fig. 2 will be below the sensitivity curves of BBO and DECIGO. XTE J1810–197, SGR 0501+4516, and 1E 1048.1–5937, for example, will not be detected with this κ value, whereas the sources 1E 1547.0–5408 and SGR 1806–20 must have mass $M_{WD} \lesssim 1.3 M_{\odot}$ to be seen by the two spacial detectors. As for the other detectable sources in Fig. 2, they cannot be too massive ($\gtrsim 1.2 M_{\odot}$) to continue being observed by BBO.

5 SUMMARY

Besides the search and detection of GWs from the merger events (Abbott et al. 2017c, 2019), the search for continuous GWs has been of great interest in the scientific community. It is well known that, besides compact binaries, rapidly rotating NSs are promising sources of GWs, which could be detected in a near future by Advanced LIGO (aLIGO) and Advanced Virgo (AdV), and also by the planned Einstein Telescope (ET) and the space-based LISA, BBO, and DECIGO. These sources generate continuous GWs whether they are not perfectly symmetric around their rotation axis, i.e. if they present some equatorial ellipticity. Undoubtedly, SGRs and AXPs are also good candidates in this context. Here we investigate the gravitational radiation from these objects described as fast-spinning WDs using the magnetic deformation mechanism. It is worth stressing that these putative uncommon WDs are known to have a high rotation (a few seconds to minutes) and a huge magnetic field (10^6 – 10^{10} G).

Then, by describing SGRs/AXPs as rotation-powered WD pulsars, we consider the role played by the magnetic dipole field on the deformation of these objects and its consequences as regards the generation of GWs considering a mass range $1.0 M_{\odot} \leq M_{WD} \leq 1.4 M_{\odot}$ for these sources. It is worth mentioning that this is the first time in the literature that the GW counterparts for SGRs/AXPs are modelled as fast-spinning and magnetized WDs.

We note that some SGRs/AXPs, described as fast-spinning and massive WDs, emit GWs with amplitudes that could be detected by BBO and DECIGO, namely, 1E 1547.0–5408 and SGR 1806–20 can be observed for the entire considered mass range for 1 yr of observation time, while SGR 1900+14, CXOU J171405.7–381031, Swift J1834.9–0846, SGR 1627–41, PSR J1622–4950, SGR J1745–2900, and SGR 1935+2154 can be detected for the entire considered mass range for 5 yr of observation time. The sources XTE J1810–197, SGR 0501+4516, and 1E 1048.1–5937, in turn, can also be observed if they do not have so large masses.

Last but not least, it is worth mentioning that recent astronomical observations suggest that we should revisit the real nature of AXP/SGRs: are they really magnetars or fast-spinning and magnetized WDs? Thereby, a possible detection of continuous GWs coming from SGRs/AXPs would be a good indication that could corroborate the WD model, because for the NSs' description, they are far below the BBO and DECIGO sensitivity curves. We also encourage future observational campaigns to determine the radii and the magnetic field of these sources to elucidate the real nature of SGRs and AXPs.

ACKNOWLEDGEMENTS

We thank the anonymous referee for useful criticisms and suggestions which have improved our paper. MFS thanks CAPES for

the financial support. JGC is likewise grateful to the support of CNPq (421265/2018-3 and 305369/2018-0). JCNA thanks FAPESP (2013/26258-4) and CNPq (308367/2019-7) for partial financial support.

DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.

REFERENCES

- Abbott B. P. et al., 2016, *Phys. Rev. Lett.*, 116, 061102
 Abbott B. P. et al., 2017a, *Phys. Rev. Lett.*, 118, 221101
 Abbott B. P. et al., 2017b, *Phys. Rev. Lett.*, 119, 141101
 Abbott B. P. et al., 2017c, *Phys. Rev. Lett.*, 119, 161101
 Abbott B. P. et al., 2017d, *ApJ*, 851, L35
 Abbott B. P. et al., 2019, *ApJ*, 882, L24
 Althaus L. G., García-Berro E., Isern J., Córscico A. H., 2005, *A&A*, 441, 689
 Althaus L. G., García-Berro E., Isern J., Córscico A. H., Rohrmann R. D., 2007, *A&A*, 465, 249
 Amaro-Seoane P. et al., 2017, preprint (arXiv:1702.00786)
 Barstow M. A., Jordan S., O'Donoghue D., Burleigh M. R., Napiwotzki R., Harrop-Allin M. K., 1995, *MNRAS*, 277, 971
 Bonazzola S., Gourgoulhon E., 1996, *A&A*, 312, 675
 Borges S. V., Rodrigues C. V., Coelho J. G., Malheiro M., Castro M., 2020, *ApJ*, 895, 26
 Boshkayev K., Izzo L., Rueda J. A., Ruffini R., 2013a, *A&A*, 555, A151
 Boshkayev K., Rueda J. A., Ruffini R., Siutsou I., 2013b, *ApJ*, 762, 117
 Cáceres D. L., de Carvalho S. M., Coelho J. G., de Lima R. C. R., Rueda J. A., 2017, *MNRAS*, 465, 4434
 Camisassa M. E. et al., 2019, *A&A*, 625, A87
 Castanheira B. G., Kepler S. O., Kleinman S. J., Nitta A., Fraga L., 2013, *MNRAS*, 430, 50
 Chandrasekhar S., Fermi E., 1953, *ApJ*, 118, 116
 Coelho J. G., Malheiro M., 2014, *PASJ*, 66, 14
 Coelho J. G., Marinho R. M., Malheiro M., Negreiros R., Cáceres D. L., Rueda J. A., Ruffini R., 2014, *ApJ*, 794, 86
 Curd B., Gianninas A., Bell K. J., Kilic M., Romero A. D., Allende Prieto C., Winget D. E., Winget K. I., 2017, *MNRAS*, 468, 239
 De Araujo J. C. N., Coelho J. G., Costa C. A., 2016a, *Eur. Phys. J. C*, 76, 481
 De Araujo J. C. N., Coelho J. G., Costa C. A., 2016b, *ApJ*, 831, 35
 De Araujo J. C. N., Coelho J. G., Costa C. A., 2016c, *J. Cosmology Astropart. Phys.*, 2016, 023
 De Araujo J. C. N., Coelho J. G., Costa C. A., 2017, *Eur. Phys. J. C*, 77, 350
 De Araujo J. C. N., Coelho J. G., Ladislau S. M., Costa C. A., 2019, preprint (arXiv:1906.00774)
 Esposito P. et al., 2011, *MNRAS*, 416, 205
 Ferraro V. C. A., 1954, *ApJ*, 119, 407
 Franzon B., Schramm S., 2017, *MNRAS*, 467, 4484
 Gao H., Cao Z., Zhang B., 2017, *ApJ*, 844, 112
 Gentile Fusillo N. P. et al., 2018, *MNRAS*, 482, 4570
 Harry G. M., Fritschel P., Shaddock D. A., Folkner W., Phinney E. S., 2006, *Class. Quantum Gravity*, 23, 4887
 Hermes J. J., Kepler S. O., Castanheira B. G., Gianninas A., Winget D. E., Montgomery M. H., Brown W. R., Harrold S. T., 2013, *ApJ*, 771, L2
 Ikhshanov N. R., 1998, *A&A*, 338, 521
 Jiménez-Esteban F. M., Torres S., Rebassa-Mansergas A., Skorobogatov G., Solano E., Cantero C., Rodrigo C., 2018, *MNRAS*, 480, 4505
 Kalita S., Mukhopadhyay B., 2019, *MNRAS*, 490, 2692
 Kawamura S. et al., 2006, *Class. Quantum Gravity*, 23, S125
 Kawka A., Vennes S., Schmidt G. D., Wickramasinghe D. T., Koch R., 2007, *ApJ*, 654, 499
 Kepler S. O., Kleinman S. J., Pelisoli I., Peçanha V., Diaz M., Koester D., Castanheira B. G., Nitta A., 2010, in Werner K., Rauch T., eds, *AIP Conf. Ser.*, Vol. 1273, 17th European White Dwarf Workshop. AIP, New York, p. 19

- Kepler S. O. et al., 2013, *MNRAS*, 429, 2934
 Kepler S. O. et al., 2015, *MNRAS*, 446, 4078
 Konno K., Obata T., Kojima Y., 2000, *A&A*, 356, 234
 Kozlova A. et al., 2016, *MNRAS*, 460, 2008
 Külebi B., Jordan S., Euchner F., Gänsicke B. T., Hirsch H., 2009, *A&A*, 506, 1341
 Külebi B., Jordan S., Nelan E., Bastian U., Altmann M., 2010, *A&A*, 524, A36
 Liebert J., Schmidt G. D., Green R. F., Stockman H. S., McGraw J. T., 1983, *ApJ*, 264, 262
 Lobato R. V., Malheiro M., Coelho J. G., 2016, *Int. J. Mod. Phys. D*, 25, 1641025
 Lobato R. V., Coelho J. G., Malheiro M., 2017, *J. Phys: Conf. Ser.*, 861, 012005
 Lopes de Oliveira R., Bruch A., Rodrigues C. V., Oliveira A. S., Mukai K., 2020, *ApJ*, 898, L40
 Maggiore M., 2008, *Gravitational Waves: Volume 1: Theory and Experiments*. Oxford Univ. Press, Oxford
 Malheiro M., Rueda J. A., Ruffini R., 2012, *PASJ*, 64, 56
 Marsh T. R. et al., 2016, *Nature*, 537, 374
 Mereghetti S., Tiengo A., Esposito P., La Palombara N., Israel G. L., Stella L., 2009, *Science*, 325, 1222
 Mukhopadhyay B., Rao A. R., 2016, *J. Cosmology Astropart. Phys.*, 5, 007
 Mukhopadhyay B., Rao A. R., Bhatia T. S., 2017, *MNRAS*, 472, 3564
 Olausen S., Kaspi V., 2014, *ApJS*, 212, 6
 Pereira J. P., Coelho J. G., de Lima R. C. R., 2018, *Eur. Phys. J. C*, 78, 361
 Regimbau T., de Freitas Pacheco J. A., 2006, *A&A*, 447, 1
 Robson T., Cornish N. J., Liug C., 2019, *Class. Quantum Gravity*, 36, 105011
 Rueda J. A., Boshkayev K., Izzo L., Ruffini R., Lorén-Aguilar P., Külebi B., Aznar-Siguán G., García-Berro E., 2013, *ApJ*, 772, L24
 Schmidt G. D., West S. C., Liebert J., Green R. F., Stockman H. S., 1986, *ApJ*, 309, 218
 Schmidt G. D., Bergeron P., Liebert J., Saffer R. A., 1992, *ApJ*, 394, 603
 Shapiro S. L., Teukolsky S. A., 1983, *Black Holes, White Dwarfs and Neutron Stars: The Physics of Compact Objects*. John Wiley & Sons, New York
 Sousa M. F., Coelho J. G., de Araujo J. C. N., 2020, *MNRAS*, 492, 5949
 Terada Y. et al., 2008a, *Adv. Space Res.*, 41, 512
 Terada Y. et al., 2008b, *PASJ*, 60, 387
 Usov V. V., 1988, *Pisma v Astronomicheskii Zhurnal*, 14, 606
 Yagi K., Seto N., 2011, *Phys. Rev. D*, 83, 044011
 Yagi K., Seto N., 2017, *Phys. Rev. D*, 95, 109901

This paper has been typeset from a $\text{\TeX}/\text{\LaTeX}$ file prepared by the author.