Comparing the adjoint- and ensemble-based approaches to observation impact on short-range forecasts

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Outline

FSOI at GMAO: from Adjoint- to Ensemble-based?

Evolution of Forecast Sensitivity and Observation Impact (FSOI) at GMAO:

- GMAO has been calculating FSOI in its Forward Processing (FP) system for several years.
- FP has evolved from 3dVar to Hybrid-3dVar to what is presently Hybrid-4dEnVar.
- Our strategy follows the Langland & Baker (2004) approach and relies on the availability of an adjoint model.
- Along the years the GMAO forward model has gone from FV to FV^3; accordingly, the adjoint model has gone from AD-FV to AD-FV^3.
- Linearized physics, and corresponding adjoint, has evolved from simple diffusion and vertical drag to more elaborate accountability of convection (Holdaway, Errico, Gelaro & Kim).

Ensemble DA opens the door to bypass the Adjoint Model:

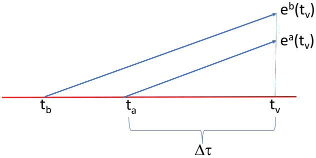
- In a dual-analysis system (Var & Ens) the possibility exists to base FSOI fully on the ensemble - this has its caveats (see what follows).
- Alternatively:

Method I The AD-Var-analysis can be adapted to make use of an ensemble forecast to implicitly estimate forecast sensitivities;

Method II Or, similarly, but not identically, the ensemble might be used to explicitly estimate forecast sensitivities required by the AD-Var-analysis.

This presentation provides insights from preliminary evaluation of these possibilities.

Error reduction measure and FSOI



Forecast error:

$$e^s(t_v|t_0) = < [\mathbf{x}^f(t_v|t_0) - \mathbf{x}^v(t_v)]^T \, \mathbf{T} \, [\mathbf{x}^f(t_v|t_0) - \mathbf{x}^v(t_v)] >$$

The impact of observations is typically evaluated by studying how the error measure above changes as a consequence of assimilating observations. Whether based on adjoint or ensemble techniques, these methods require evaluation of expressions of the form:

$$\delta e \approx < \mathbf{d}^T \mathbf{K}^T \mathbf{g}_0 >$$

with **d** and **K** being the background residual vector and the analysis gain matrix, and **g** amounting to a forecast sensitivity vector whose approximation leads to all kinds of formula.

	AD-Solver ($\mathbf{K}^T$)	Forecast Sensitivity ($\mathbf{g}_0$)	This Talk
VA-FSOI	Var	ADM	done
EE-FSOI	En	En	done
VE-FSOI	Var	En	done
EA-FSOI	En	ADM	—

VA-FSOI vs EE-FSOI in a Dual-Analysis Hybrid System

Adjoint- and Ensemble-based FSOI

Variational-Adjoint-FSOI (VA-FSOI)

Second-order Approximation (Trapezoidal rule; Langland & Baker 2004; Tellus):

$$\mathbf{g}_0^A \equiv \frac{1}{2}(\mathbf{M}_a^T \mathbf{T}\mathbf{e}^a + \mathbf{M}_b^T \mathbf{T}\mathbf{e}^b).$$

And in a system such as GSI, the calculation of δe can be done as in:

$$\delta e \approx < \mathbf{d}^T \mathbf{R}^{-1} \mathbf{H}\tilde{\mathbf{g}} >,$$

where $\tilde{\mathbf{g}}$ is derived from the GSI-hybrid solver as in (double-CG or Bi-CG):

$$\begin{aligned} (\mathbf{B} + \mathbf{B}\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\mathbf{B})\mathbf{z} &= \mathbf{B}\mathbf{g}_0^A \\ \tilde{\mathbf{g}} &= \mathbf{B}\mathbf{z} \end{aligned}$$

for $\mathbf{B} = \beta_c \mathbf{B}_c + \beta_e \mathbf{B}_e$.

Ensemble-Ensemble-FSOI (EE-FSOI)

In Ensemble systems, the gradient is defined with respect to the ensemble mean:

$$\mathbf{g}_0^E \equiv \frac{1}{2}\mathbf{X}_a^{fT}\mathbf{T}(\bar{\mathbf{e}}^a + \bar{\mathbf{e}}^b),$$

where $\mathbf{X}_a^f \equiv \mathbf{X}^f(t_v|t_a)$ is a matrix created from the ensemble perturbation of forecasts issued from t_a and valid at t_v, and the over-bar represents ensemble average.

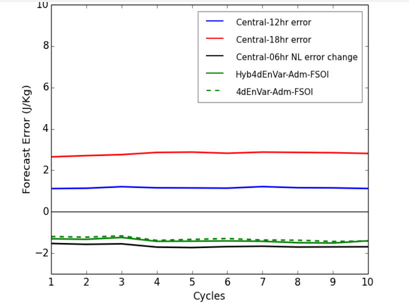
And in a system such as the EnSRF, calculation of δe amount to:

$$\delta e \approx \frac{1}{2} < \mathbf{d}^T \mathbf{R}^{-1} \mathbf{H}\,(\mathbf{L} \bullet \mathbf{X}^a \mathbf{X}^{fT})\,\mathbf{T}(\bar{\mathbf{e}}^a + \bar{\mathbf{e}}^b) >,$$

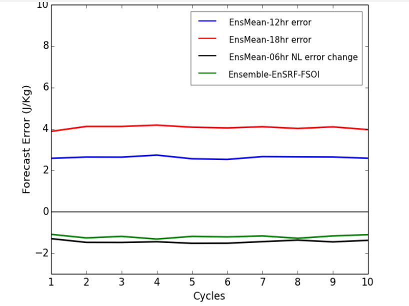
where $\mathbf{X}_a \equiv \mathbf{X}_a(t_a)$ is a matrix formed from ensemble analysis perturbations (Kalnay *et al.*, 2012, Tellus; Ota *et al.*, 2013, Tellus). An argument has been made to have $\mathbf{L}$ above as an advected form of the $\mathbf{L}$ used in the forward ensemble analysis.

VA-FSOI vs EE-FSOI

From Central Forecasts



From Ensemble Forecasts



- Error reductions are similar between central and ensemble forecasts, thought latter is slightly smaller in absolute value for 12-hour forecasts.
- Left: compares FSOI when backward Var changed from Hyb-4dEnVar to 4dEnVar.
- Right: presents EE-FSOI.

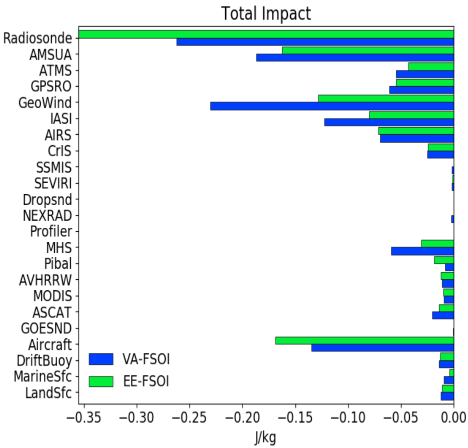
VA-FSOI vs EE-FSOI

Total Impact

- To render a fair comparison, we calculate Adjoint-based impacts for a changed Adjoint analysis integration where the climatological term is shut off, thus converting the backward run into a 4dEnVar instead of its default (FP-like) Hybrid-4dEnVar.
- At first glance, impact rankings are similar.
- Closer examination reveals considerable differences (*e.g.*, radiosondes and satellite winds).
- Differences are also non-negligible for MW and IR satellite radiances (AMSU-A, ATMS, IASI).

Overall this comparison reflects that:

- Ensemble mean forecasts are unrelated to the central forecast.
- But more importantly, the ensemble analysis handles observations largely differently to how the hybrid analysis does.

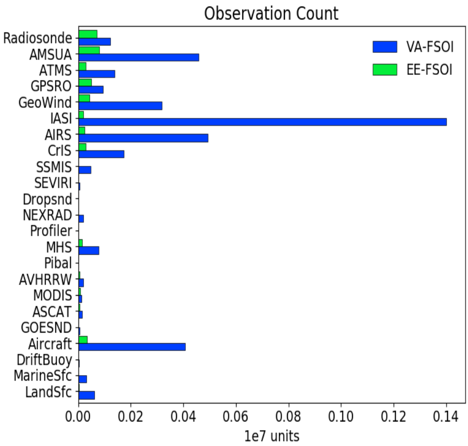


VA-FSOI vs EE-FSOI

Observation Counts

- The difference in treatment of observations between ensemble and central analyses is evidenced in the observation count.
- The GSI and EnSRF solvers have considerably different convergence criteria.
- Even with the ideal DFS-based criterium (chosen here), the EnSRF ignores a very large percentage of the observations.

All-in-all we don't think observation impacts derived from the EnSRF solver represent well how the deterministic (central) analysis system uses observations.



VA-FSOI vs VE-FSOI in a Dual-Analysis Hybrid System

VA-FSOI vs VE-FSOI

Variational-Ensemble-FSOI (VE-FSOI) Method I

As in Buehner *et al.* (2018, MWR), in a EnVar, such that $\mathbf{B} = \mathbf{B}_e$, the ensemble background covariance allows for the following to be written

$$\begin{aligned} \mathbf{B}_e\mathbf{g}_0^A &= \mathbf{L} \bullet \mathbf{X}_b(\mathbf{X}_b)^T\mathbf{g}_0^A \\ &\approx \mathbf{L} \bullet \mathbf{X}_b\mathbf{X}_a^T\mathbf{M}^T(t_a, t_b)\mathbf{g}_0^A \end{aligned}$$

We can replace $\mathbf{g}_0^A$ with $\mathbf{g}_0^E$ (using central forecast errors) in the RHS to get

$$\begin{aligned} \mathbf{B}_e\mathbf{g}_0^A &\approx \mathbf{B}_e\mathbf{g}_0^E \\ &\approx \frac{1}{2}\mathbf{L} \bullet \mathbf{X}_b\mathbf{X}_a^T\mathbf{M}^T(t_a, t_b)\mathbf{X}_a^{fT}\mathbf{T}(\mathbf{e}^a + \mathbf{e}^b) \\ &= \frac{1}{2}\mathbf{L} \bullet \mathbf{X}_b\mathbf{X}_b^{fT}\mathbf{T}(\mathbf{e}^a + \mathbf{e}^b) \end{aligned}$$

which amounts to a simple change to the RHS of the minimization problem solved for calculation of observation impacts in the Var system.

Note: $\mathbf{e}^a(\mathbf{e}^b)$ replaces $\bar{\mathbf{e}}^a(\bar{\mathbf{e}}^b)$

Variational-Ensemble-FSOI (VE-FSOI) Method II

Alternatively, we can try to use the approach of Ancell & Hakim (2007; MWR) to estimate forecast sensitivities using an ensemble of forecasts.

In this case, the forecast sensitivity is estimated as in:

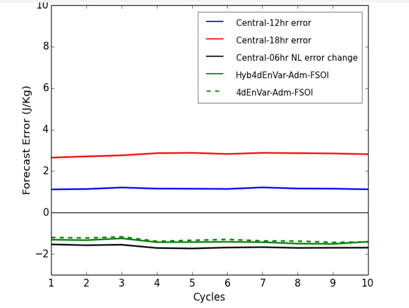
$$\frac{\partial f}{\partial \mathbf{x}} = \mathbf{D}^{-1} \begin{bmatrix} \delta\mathbf{x}_1\delta\mathbf{e}_1^T \\ \delta\mathbf{x}_2\delta\mathbf{e}_2^T \\ \vdots \\ \delta\mathbf{x}_n\delta\mathbf{e}_n^T \end{bmatrix}$$

where $\mathbf{D} = diag(||\delta\mathbf{x}_1||^2, ||\delta\mathbf{x}_2||^2, \cdots, ||\delta\mathbf{x}_n||^2)$, $dim(\mathbf{x}_i) = dim(\mathbf{e}_i) = M \times 1$, and n is the state-space dimension.

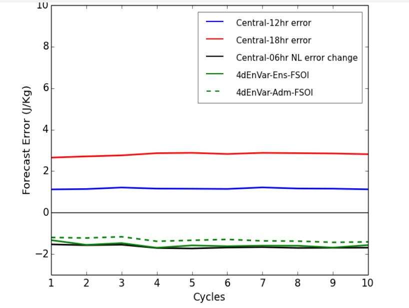
This presentation will not present results for Method II.

VA-FSOI vs VE-FSOI Method I

From Central Forecasts



From Central Forecasts



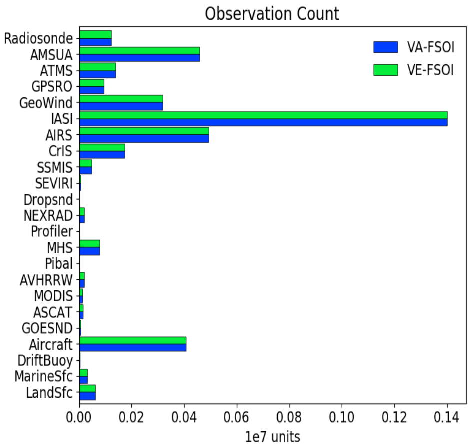
- Lack of advection of localization scales in the RHS of the Var impact expression motivates following Buehner *et al.* (2018; MWR) and evaluating 12-hr instead of 24-hr FSOI.
- Left: compares FSOI when backward Var changed from Hyb-4dEnVar to 4dEnVar.
- Right: compares VA-FSOI with VE-FSOI Method I.

Remark: 32-member ensemble perturbations seem rather reasonable replacement for ADM for 12-hr sensitivity calculation in 4dEnVar context.

VA-FSOI vs VE-FSOI Method I

Observation Counts

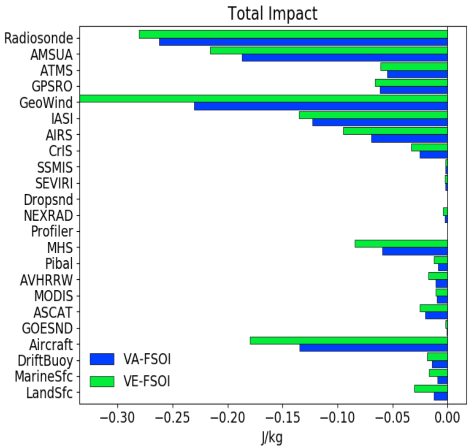
- The two approaches treat the observations in exactly the same way, and fully consistent with how the forward (Hyb-4dEnVar) solver treats them.
- Replacing the Adjoint Model with Ensemble perturbations to estimate forecast sensitivities leaves the analysis solver untouched wrt each other.
- The figure on the right shows observation counts between the VA-FSOI and VE-FSOI Method I techniques, for backward integrations of 4dEnVar, covering a 10-day period.



VA-FSOI vs VE-FSOI Method I

Total Impact

- Overall, impacts don't seem to change much and are largely comparable when ADM is replaced with Ensemble perturbations.
- Closer look reveals satellite winds (GeoWind), MW sensors (MHS, AMSU-A), aircrafts and near surface observations to have larger impact when Ens-Perts are used compared to when ADM is used to estimate forecast sensitivity.
- The above seems to be consistent with the fact that the simply parameterized adjoint physics is expected to mis-represent water and near surface fields as compared to the full GCM.



Closing Remarks

- More importantly, though the observation operators in the hybrid GSI and EnSRF are shared, GSI and EnSRF treat observations rather differently.
- In a dual hybrid DA system, when a (low resolution) ensemble analysis filter is used to provide flow dependence to a (high resolution) hybrid analysis, certain configurations of the ensemble filter might discourage assessing observation impact using the EE-FSOI based on the EnDA part of system.
- The comment above applies particularly to GSI-EnSRF-based systems.
- As in Buehner *et al.*, we have shown that it is possible to enable the Var system to derive observation impacts with forecast sensitivities calculated from the ensemble thus avoiding the adjoint model.